Modul #05

TTI3J3
SISTEM KOMUNIKASI 2

M-PSK (Phase Shift Keying)
Modulasi, Demodulasi, Kinerja

Program Studi S1 Teknik Telekomunikasi
Fakultas Teknik Elektro – Telkom University
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What is Modulation?

- Encoding information in a manner suitable for transmission.
  - Translate baseband source signal to bandpass signal
  - Bandpass signal: “modulated signal”
- How?
  - Vary amplitude, phase or frequency of a carrier
- Demodulation: extract baseband message from carrier

Modulasi Analog

Persamaan sinyal pembawa /carrier :

$$Vc(t) = Vc \sin(\omega_c t + \theta)$$

Modulasi amplitude

(amplitude modulation, AM)

Modulasi sudut

(angle modulation)

$(\omega_c t + \theta)$

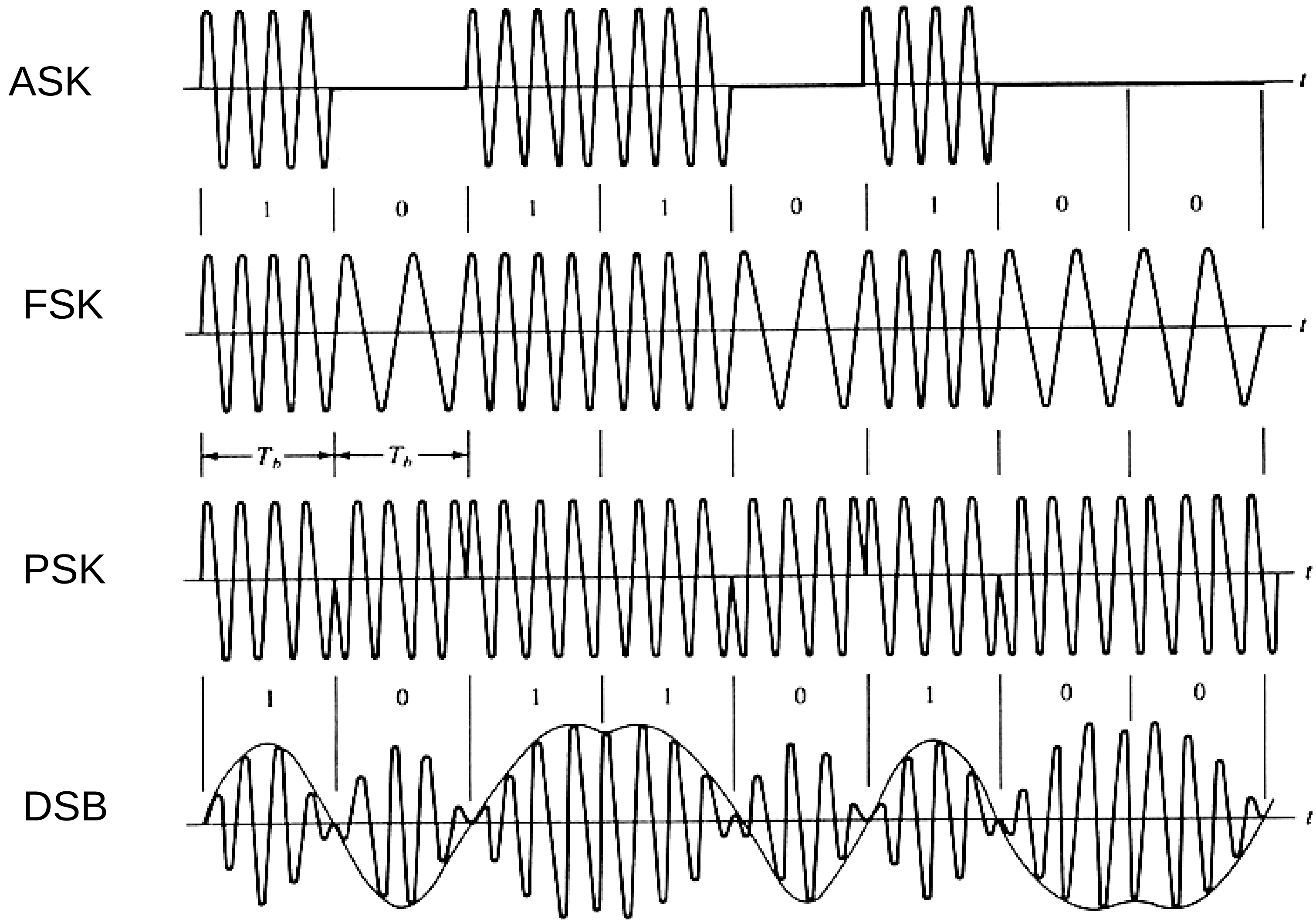
Modulasi frekuensi

(frequency modulation, FM)

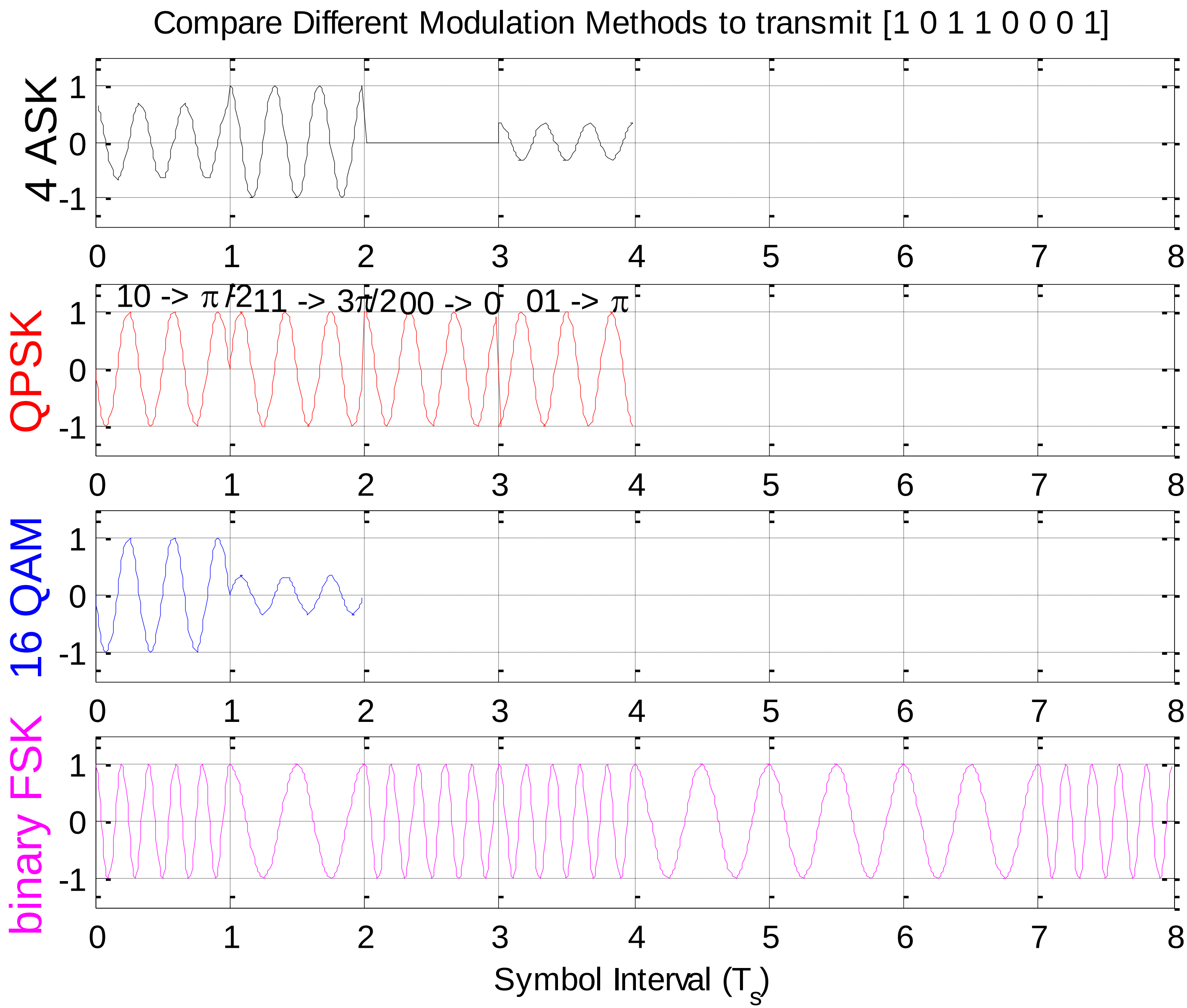
Modulasi fase

(phase modulation, PhM)

Gambar beberapa modulasi Digital



Gambar lain beberapa modulasi Digital



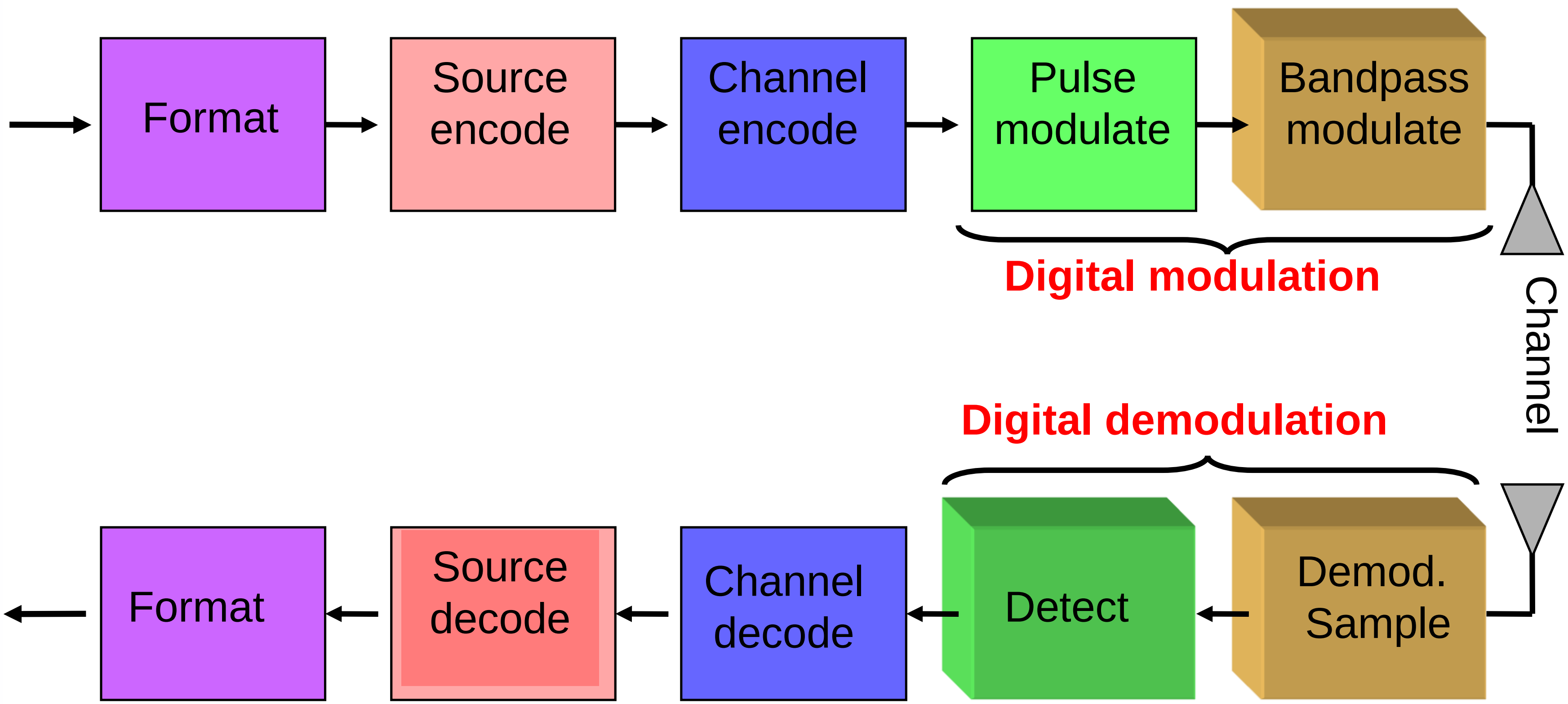
Digital vs Analog Modulation

- Cheaper, faster, more power efficient
- Higher data rates, power error correction, impairment resistance:
  - Using coding, modulation, diversity
  - Equalization, multicarrier techniques for ISI mitigation
- More efficient multiple access strategies, better security: CDMA, encryption etc

Goals of Modulation Techniques

- **High Bit Rate**
- **High Spectral Efficiency** (*max Bps/Hz*)
- **High Power Efficiency** (*min power to achieve a target BER*)
- **Low-Cost/Low-Power Implementation**
- **Robustness to Impairments**

Block diagram of a Digital Communication System

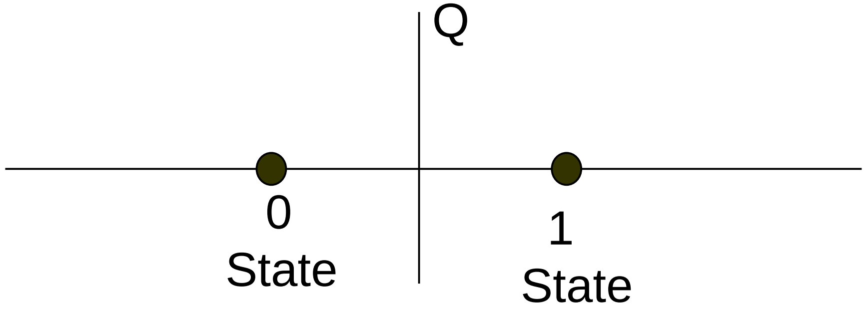


Binary Phase Shift Keying

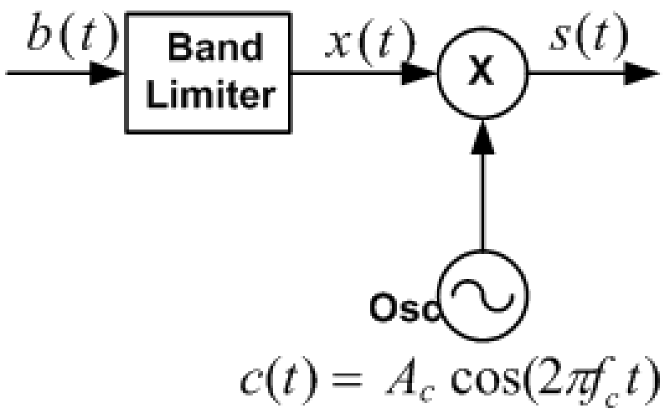
- Menggunakan alternatif-alternatif fasa gelombang sinus utk mengkodekan bit-bit:
  - Fasa dipisahkan 180 derajat.
  - Sederhana utk diimplementasikan, tidak efisien dalam penggunaan bandwidth.
  - Sangat kokoh, sering digunakan secara extensif pada komunikasi satelit.

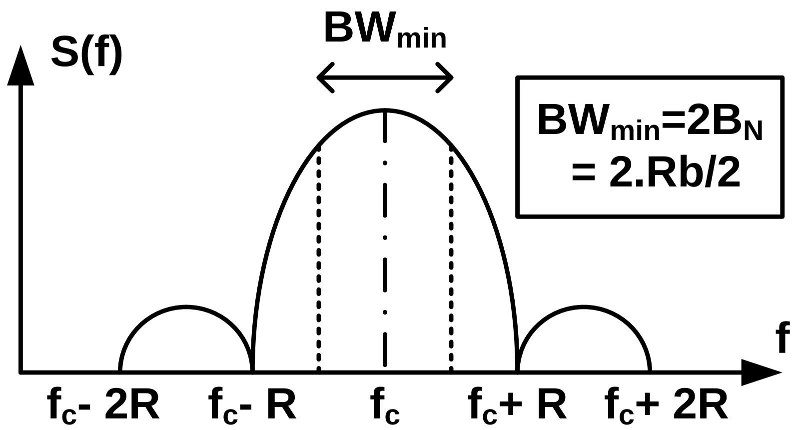
$$s_1(t) = A_c \cos(2\pi f_c t) \qquad \textbf{binary '1'}$$

$$s_2(t) = A_c \cos(2\pi f_c t + \pi) \quad \textbf{binary '0'}$$



Pembangkitan BPSK

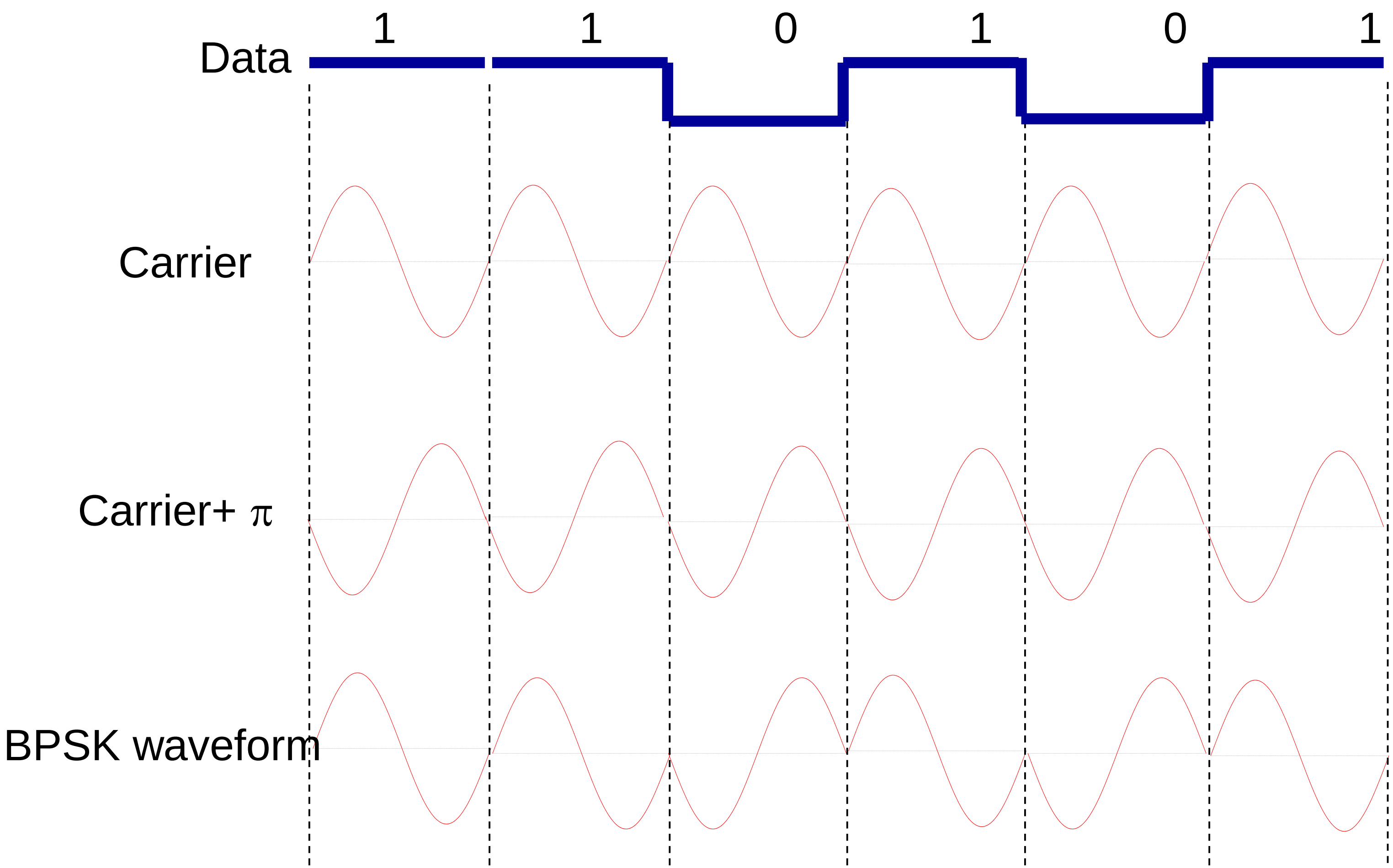




B_N=Bandwidth Nyquist

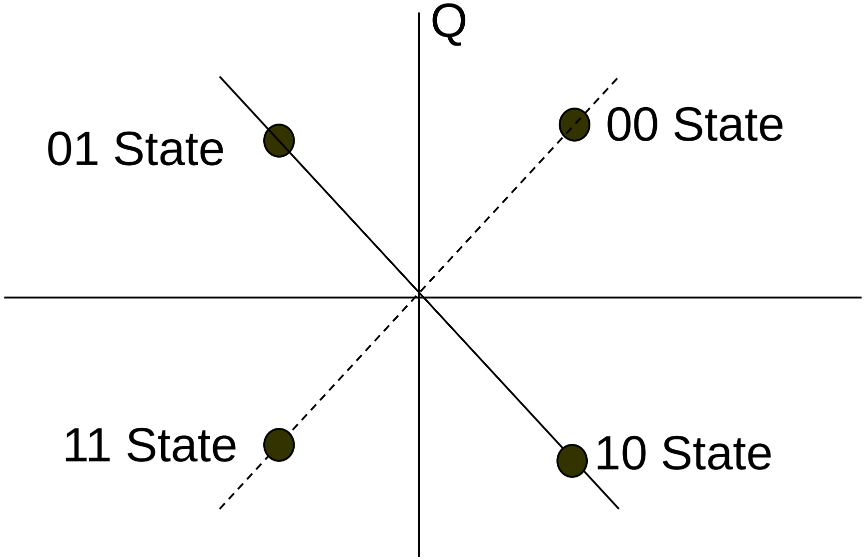
$$s(t) = \begin{cases} s_1(t) = A_c \cos(2\pi f_c t) & \text{binary '1'} \\ s_2(t) = A_c \cos(2\pi f_c t + \pi) & \text{binary '0'} \end{cases}$$

Contoh BPSK



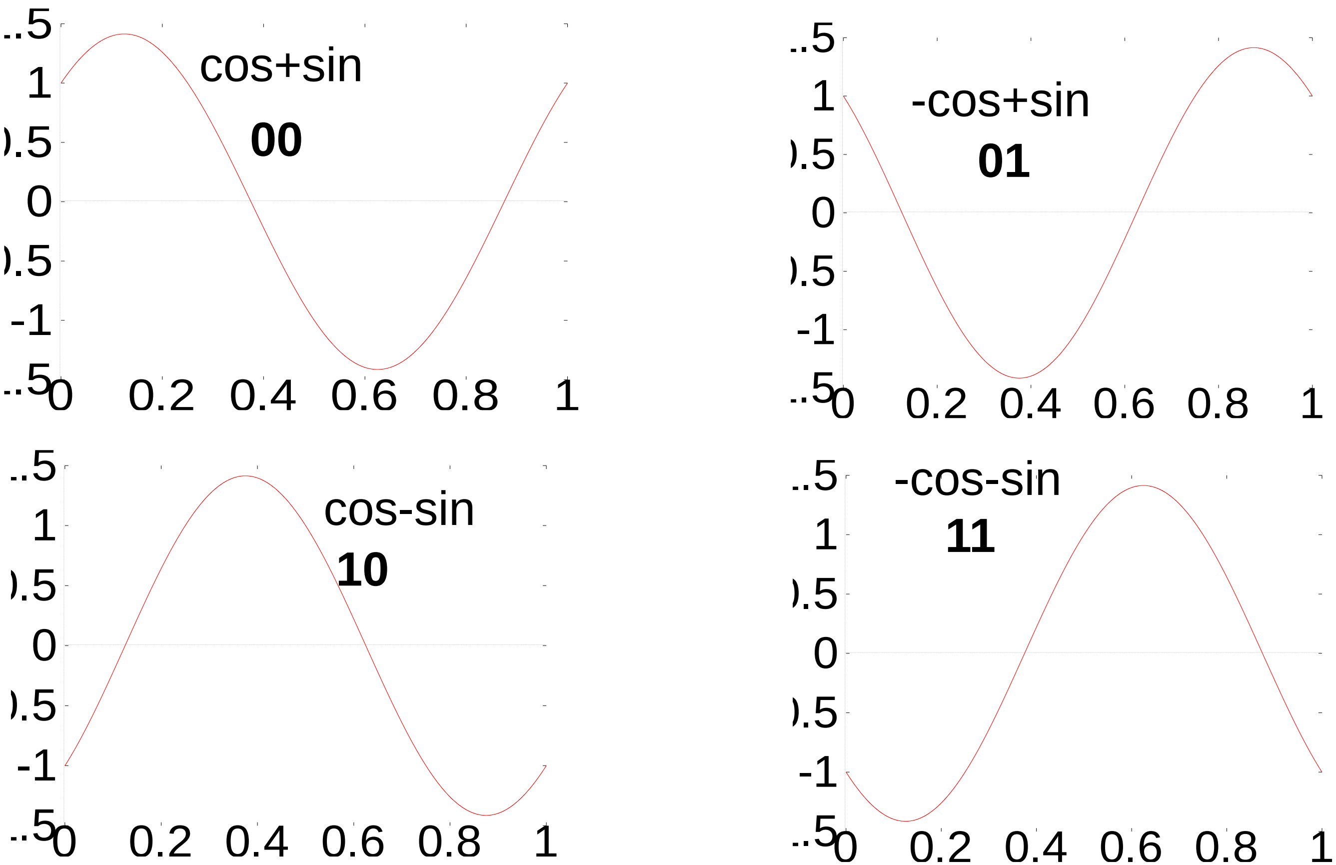
Quadrature Phase Shift Keying

- Teknik modulasi multilevel : 2 bit per symbol
- Lebih efisien spektrum, lebih kompleks receiver.
- Dua kali lebih efisien bandwidth daripada BPSK

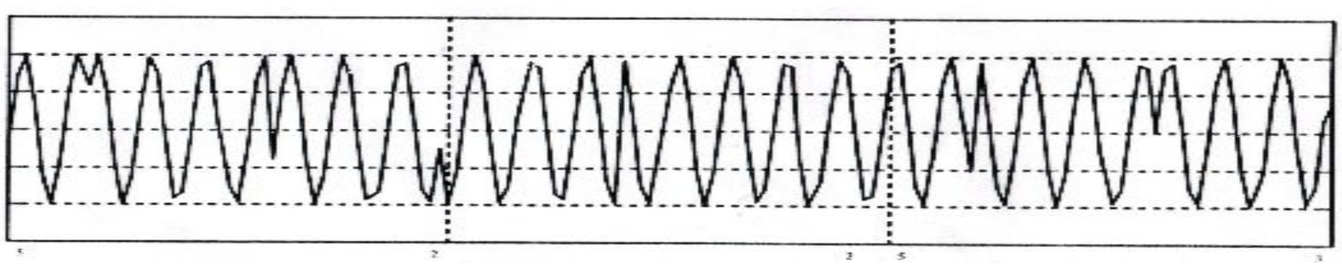


Phase of Carrier: $\pi/4$, $2\pi/4$, $5\pi/4$, $7\pi/4$

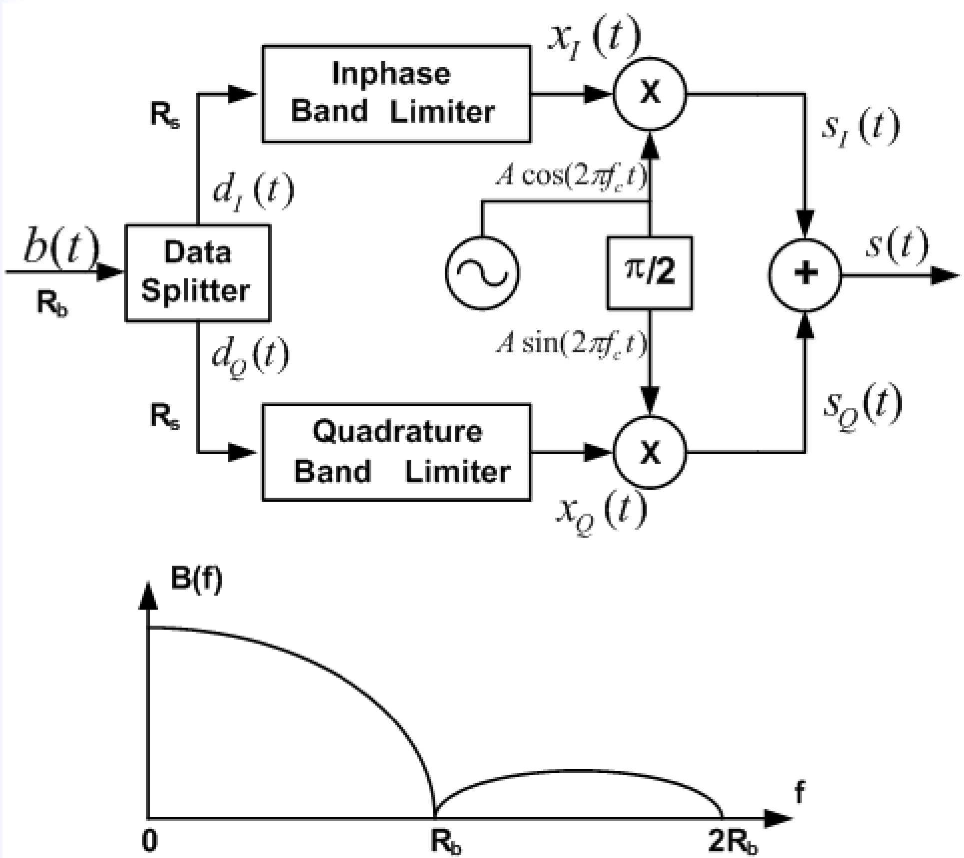
4 bentuk gelombang berbeda:

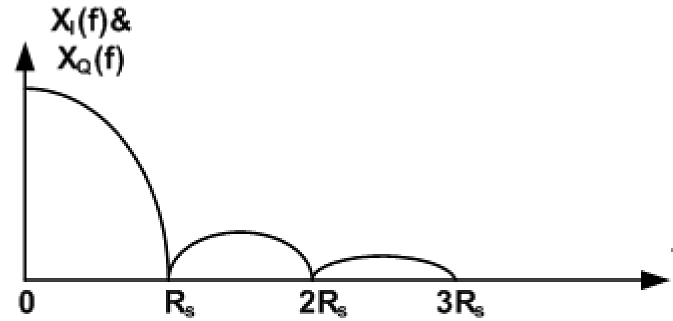


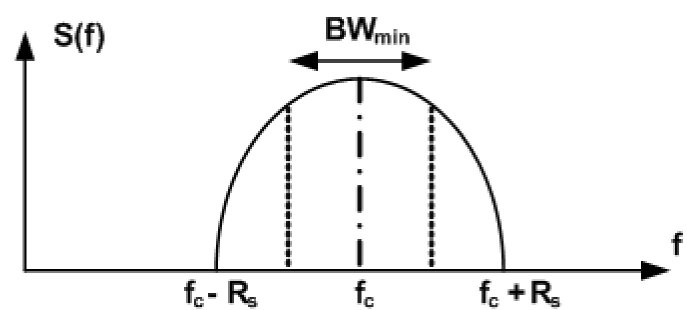
Bentuk Sinyal QPSK



Pembangkitan sinyal QPSK







- Bandpass modulation: The process of converting data signal to a sinusoidal waveform where its amplitude, phase or frequency, or a combination of them, is varied in accordance with the transmitting data.
- Bandpass signal (**General Condition**):

$$s_i(t) = h(t)\sqrt{\frac{2E_i}{T}}\cos\left(\omega_c t + (i-1)\Delta\omega t + \phi_i(t)\right) \quad 0 \le t \le T$$

where $h(t)$ is the baseband pulse shape with energy E_h.

- We assume here (otherwise will be stated):
  - $h(t)$ is a rectangular pulse shape with unit energy.
  - Gray coding is used for mapping bits to symbols.
  - E_s denotes average symbol energy given by $E_s = \frac{1}{M}\sum_{i=1}^{M} E_i$

Phase Shift Keying (PSK)

I. PSK signal waveform (transmitted signal):

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos[\omega_0 t + \phi_i(t)], \quad \text{phase: } \phi_i(t) = \frac{2\pi i}{M}, \quad 0 \le t \le T, \quad i = 1, \cdots, M,$$

- Phase (examples):

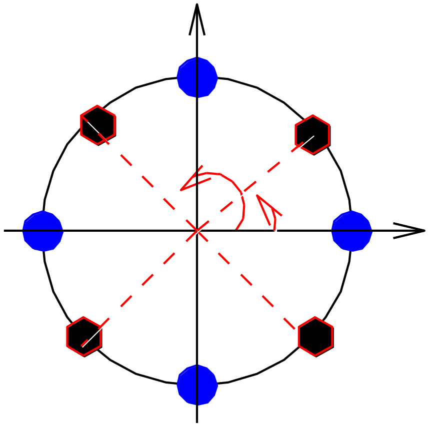
$$\text{BPSK}(M = 2): \quad \phi_i \in \{\pi, 0\}$$

$$\text{QPSK}(M = 4): \quad \phi_i \in \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 0\right\} \text{ or } \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}_{i=1/2}^{7/2}$$

- Symbol energy and symbol interval

E is symbol energy. T is symbol interval.

Can you show that $E = \int_0^T s_i^2(t)dt$?



II. Signal space representation

Note: decomposition of PSK signal waveform:

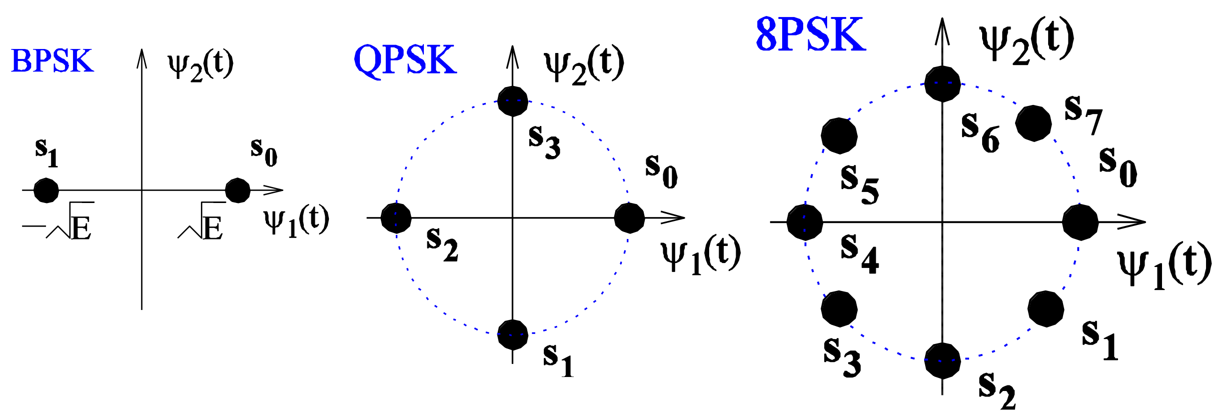
$$s_i(t) = \sqrt{\frac{2E}{T}}\cos[\phi_i(t)]\cos(\omega_0 t) - \sqrt{\frac{2E}{T}}\sin[\phi_i(t)]\sin(\omega_0 t)$$

1) Orthonormal basis: $\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_0 t), \quad \psi_2(t) = \sqrt{\frac{2}{T}}\sin(\omega_0 t)$

2) Signal vector: $s_i(t): \ \mathbf{s}_i = \left(\sqrt{E}\cos\frac{2\pi i}{M}, \ -\sqrt{E}\sin\frac{2\pi i}{M}\right), \quad i = 1, \cdots, M$

3) Constellations:

(examples)



Signal space representation (cont.)

4) A proof of signal space representation

- Bases are orthonormal

$$\int_{-\infty}^{\infty}\psi_1^2(t)dt=\int_0^T 2/T\cos^2(\omega_0 t)dt=2/T\int_0^T[1+\cos(2\omega_0 t)]/2dt=1.$$

$$\int_{-\infty}^{\infty}\psi_2^2(t)dt=\int_0^T 2/T\sin^2(\omega_0 t)dt=2/T\int_0^T[1-\cos(2\omega_0 t)]/2dt=1.$$

$$\int_{-\infty}^{\infty}\psi_1(t)\psi_2(t)dt=\int_0^T 2/T\cos(\omega_0 t)\sin(\omega_0 t)dt=2/T\int_0^T\sin(2\omega_0 t)/2dt=0.$$

- Signal space vector for each waveform $s_i(t)$

$$a_{i1}=\int_{-\infty}^{\infty}s_i(t)\psi_1(t)dt=2\sqrt{E}/\mathrm{T}\int_0^T\cos[\omega_0 t+\phi_i(t)]\cos(\omega_0 t)dt$$

$$=\sqrt{E}/\mathrm{T}\int_0^T\cos[\phi_i(t)]+\cos[2\omega_0 t+\phi_i(t)]dt=\sqrt{E}\cos[\phi_i(t)]$$

$$a_{i2}=\int_{-\infty}^{\infty}s_i(t)\psi_2(t)dt=2\sqrt{E}/\mathrm{T}\int_0^T\cos[\omega_0 t+\phi_i(t)]\sin(\omega_0 t)dt$$

$$=\sqrt{E}/\mathrm{T}\int_0^T-\sin[\phi_i(t)]+\sin[2\omega_0 t+\phi_i(t)]dt=-\sqrt{E}\sin[\phi_i(t)]$$

Signal space representation (cont.)

5) What happens if baseband pulse-shaping ***h(t)*** is considered?

- Signal waveform:

$$s_i(t) = \sqrt{\frac{2E}{T}}\, h(t) \cos[\omega_0 t + \phi_i(t)]$$

- Use basis (note that h(t) can be assumed normalized):

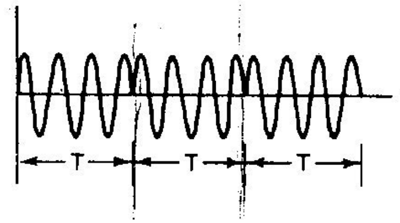
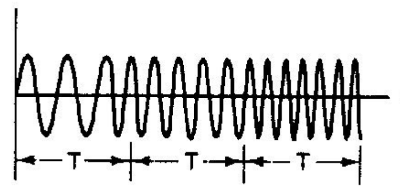
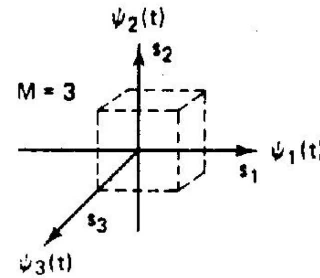
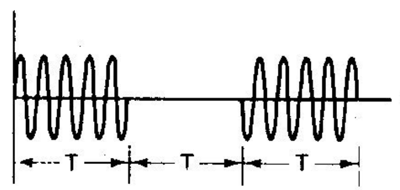
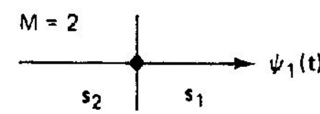
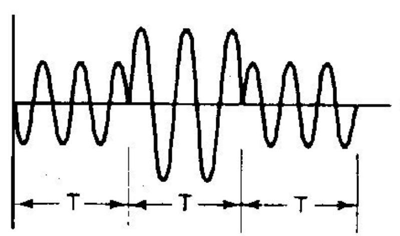
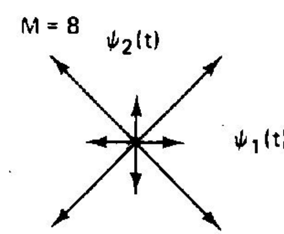
$$\psi_1(t) = \sqrt{\frac{2}{T}}\, h(t)\cos(\omega_0 t), \qquad \psi_2(t) = \sqrt{\frac{2}{T}}\, h(t)\sin(\omega_0 t)$$

- Signal space vector is still:

$$s_i(t): \quad \mathbf{s}_i = \left(\sqrt{E}\cos\frac{2\pi i}{M},\quad -\sqrt{E}\sin\frac{2\pi i}{M}\right), \quad i = 1, \cdots, M$$

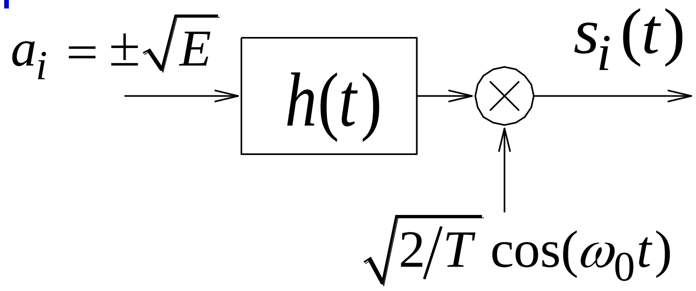
- Conclusion: there is no difference in signal space whether pulse-shaping is considered. We can study only PSK instead of the more general PM.

Signal Space of several modulation

	Analytic	Waveform	Vector
(a) PSK	$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + 2\pi i/M)$ $i = 1, 2, \ldots, M$ $0 \le t \le T$		
(b) FSK	$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi)$ $i = 1, 2, \ldots, M$ $0 \le t \le T$		
(c) ASK	$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos(\omega_0 t + \phi)$ $i = 1, 2, \ldots, M$ $0 \le t \le T$		
(d) ASK/PSK (APK)	$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos[\omega_0 t + \phi_i(t)]$ $i = 1, 2, \ldots, M$ $0 \le t \le T$		

PSK modulator

- Special case: BPSK modulator

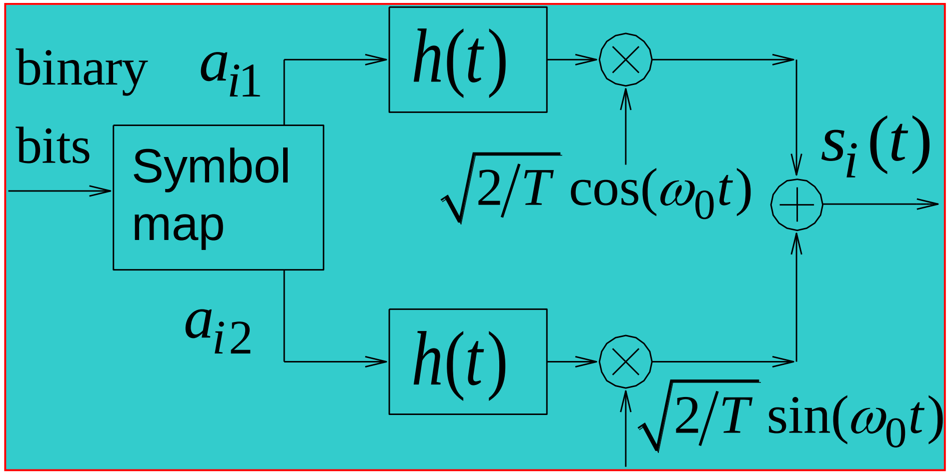


- General case: M-ary PSK modulator

Note:

Inputs are signal-space vector.

Carriers are in basis form.



$$s_i(t) = a_{i1}\sqrt{2/T}\cos(\omega_0 t) + a_{i2}\sqrt{2/T}\sin(\omega_0 t)$$

$$\mathbf{s}_i = (a_{i1}, a_{i2}) = \left(\sqrt{E}\cos(2\pi i/M),\ -\sqrt{E}\sin(2\pi i/M)\right)$$

Bandwidth of PSK signal waveform

- Just like DSB modulation: $W_{\text{PSK}} = 2W_{\text{baseband}}$

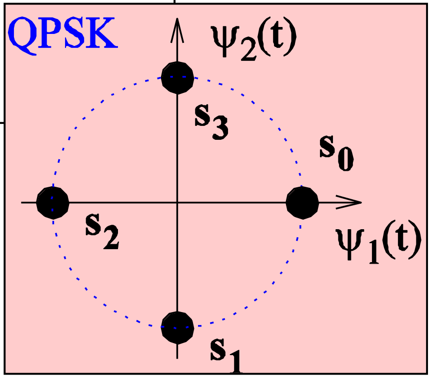
- **Exercise :** Consider QPSK transmission with date rate 2000 bps. The magnitude of the signal si(t) is √2E/T =1 volt.
  a) What is the minimum PSK signal bandwidth?
  b) Find the signal space points
  c) Draw the constellation
  d) Find signal waveform for transmitting {1001}.

a) $R_s = R_b/(\log_2 M) = 2000/2 = 1000.\ \ W_{\text{PSK}} = 2W_{\text{baseband,min}} = 2R_s/2 = 1000\text{Hz}.$

b) $\mathbf{s}_i = (\sqrt{E}\cos 2\pi i/4, -\sqrt{E}\sin 2\pi i/4)$, where $E = T/2 = 0.5\times 10^{-3}, \quad i = 1,\cdots,4$

d) Define mapping as: $\{00{:}0,\ 01{:}\pi,\ 10{:}\pi/2,\ 11{:}3\pi/2\}$.

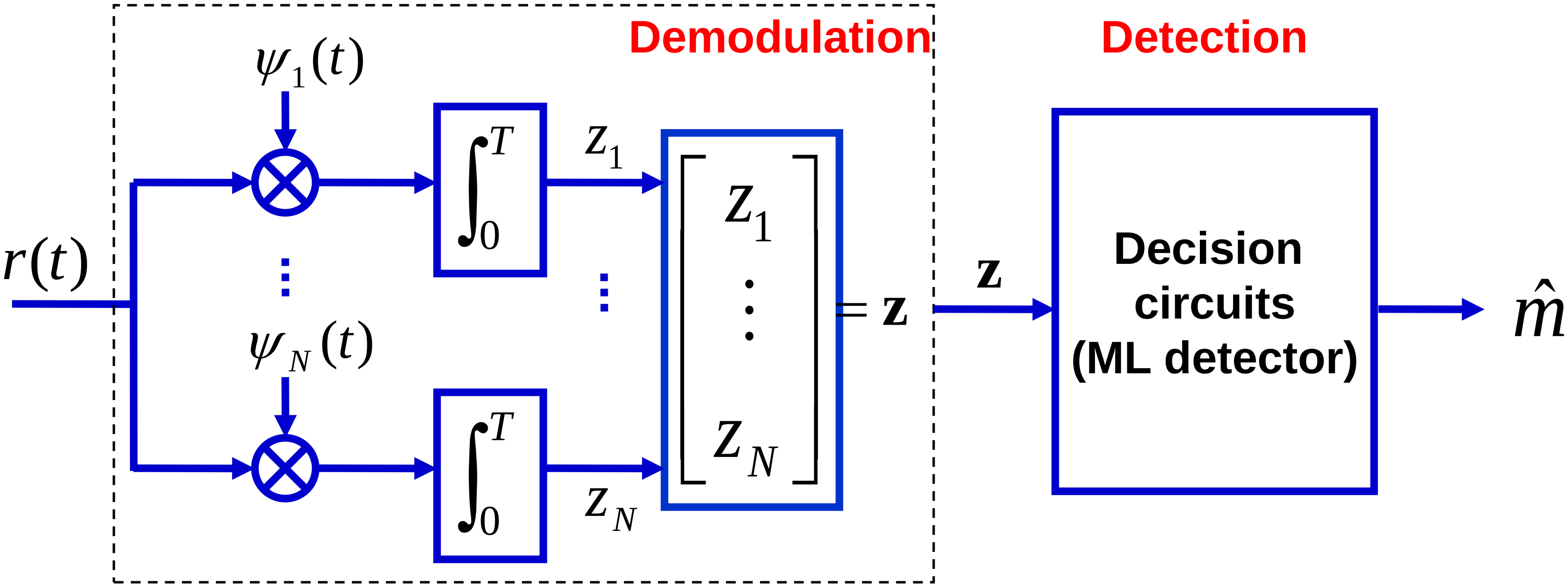
Then $\{10\} \rightarrow s_1(t) = \cos(\omega_0 t + \pi/2)$. $\{01\} \rightarrow s_2(t) = \cos(\omega_0 t + \pi)$



Phase $\phi_i(t)$ in $s_i(t)$ is different from phase of $\mathbf{s}_i$ (phase in signal space)

Demodulation and detection

- **Demodulation:** The receiver signal is converted to baseband, filtered and sampled.
- **Detection:** Sampled values are used for detection using a decision rule such as ML detection rule.



Demodulations type:

- Some notations
  - Carrier: $s(t) = A(t)\cos[\omega_0 t + \phi(t)], \qquad \omega_0 = 2\pi f_0$
  - Modulation types with respect to carrier parameters

Modulation	Varying parameter	Demodulation
PSK	$\phi(t)$	Coherent or noncoherent
QAM	both $A(t)$ and $\phi(t)$	Coherent or noncoherent
FSK	ω_0	Coherent or Noncoherent

Demodulations type:

- **Coherent detection / synchronous detection**
  - Receiver exploits knowledge of carrier's phase to detect signals
  - Require accurate phase (and frequency as well) estimation
  - Higher performance (lower error rate), but increased complexity
  - Extremely similar to baseband processing mathematically if signal space is used

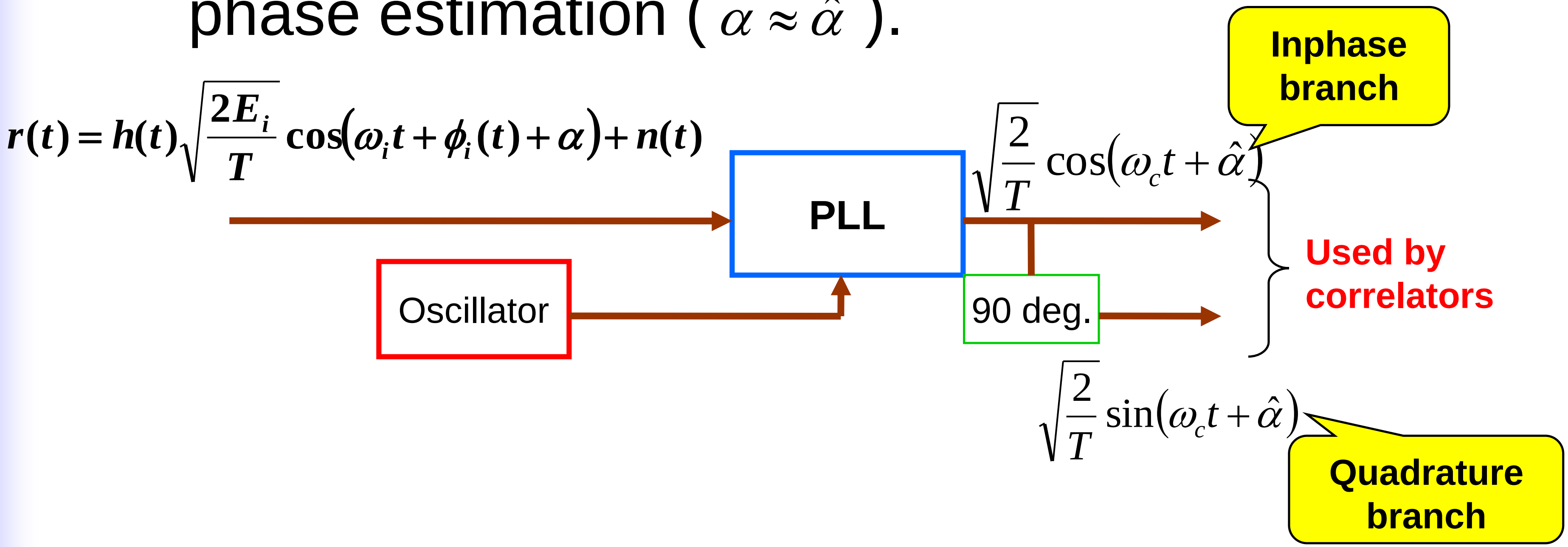
- **Noncoherent detection / asynchronous detection**
  - Receiver does not exploit carrier phase
  - Do not need accurate phase estimation
  - Reduced complexity, but lower performance (higher error rate)
  - Unique for bandpass processing: via differential encoding, or FSK energy detector

Coherent detections

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Source of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.

Coherent detection ..

- Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ($\alpha \approx \hat{\alpha}$).



Two dimensional modulation, demodulation and detection (M-PSK)

- M-ary Phase Shift Keying (M-PSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(\omega_c t + \frac{2\pi i}{M}\right)$$

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \dots, M$$

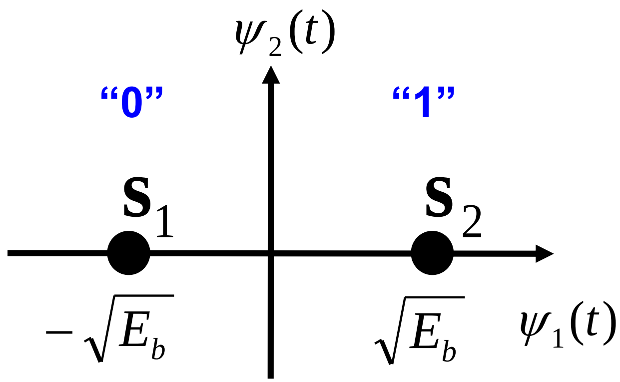
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \qquad \psi_2(t) = -\sqrt{\frac{2}{T}} \sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s} \cos\left(\frac{2\pi i}{M}\right) \qquad a_{i2} = \sqrt{E_s} \sin\left(\frac{2\pi i}{M}\right)$$

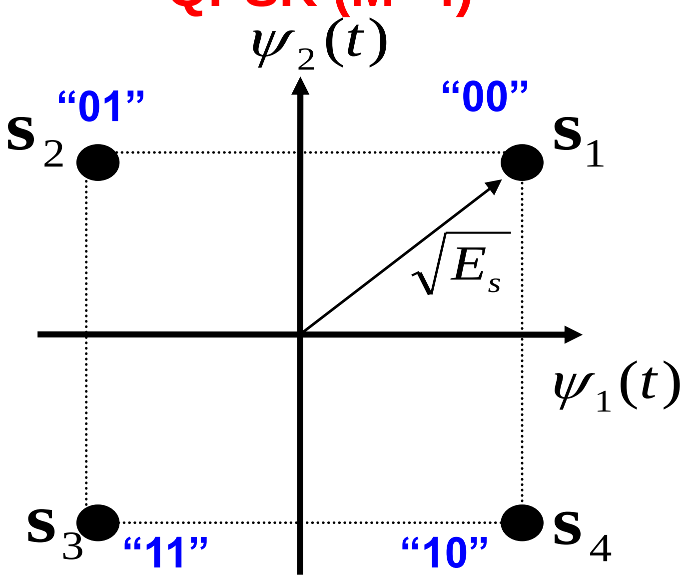
$$E_s = E_i = \left\| \mathbf{s}_i \right\|^2$$

Two dimensional mod… (MPSK)

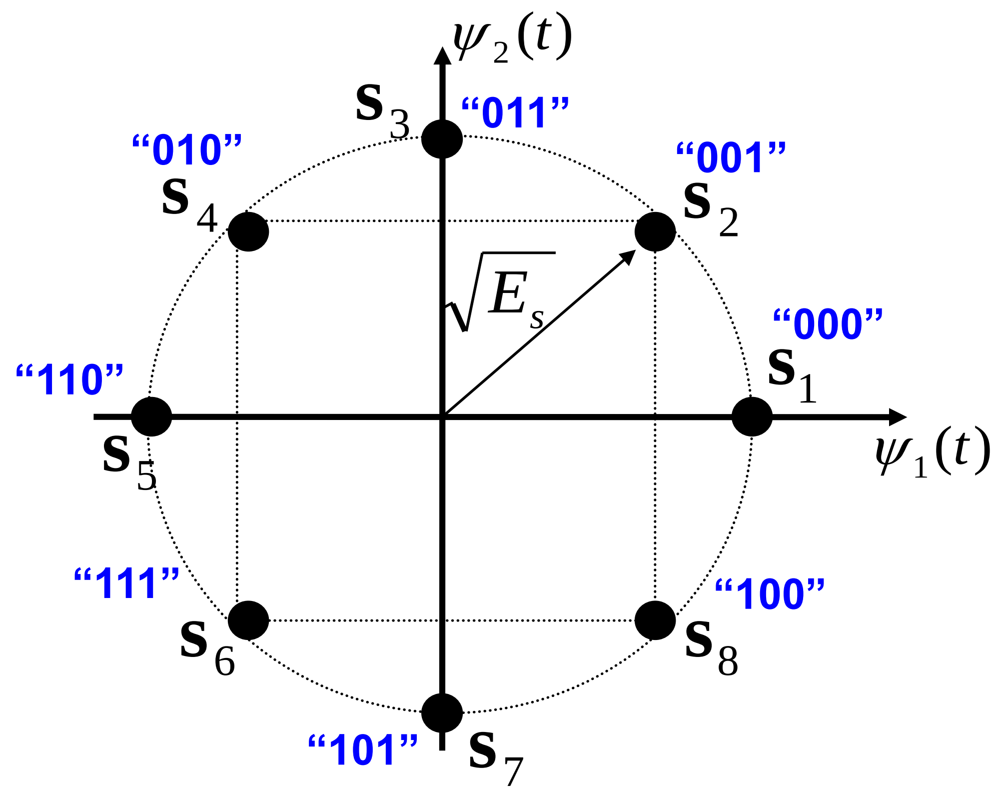
BPSK (M=2)



QPSK (M=4)

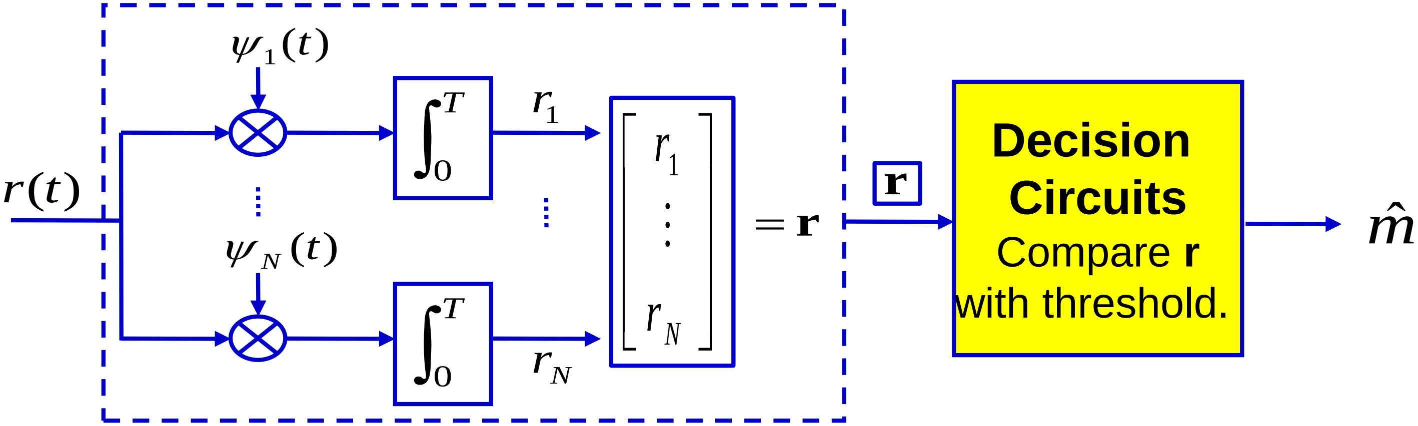


8PSK (M=8)



Error probability of bandpass modulation

- Before evaluating the error probability, it is important to remember that:
  - Type of modulation and detection (coherent or non-coherent), determines the structure of the decision circuits and hence the decision variable, denoted by z.
  - The decision variable, z, is compared with M-1 thresholds, corresponding to M decision regions for detection purposes.



Error probability …

- The matched filters output (observation vector= $\mathbf{r}$) is the detector input and the decision variable is a $z = f(\mathbf{r})$ function of $\mathbf{r}$, i.e.
  - For MPSK with coherent detection $z = \angle\mathbf{r}$ ➔ $\hat{\phi}$
  - [For non-coherent detection (M-FSK, DPSK), $z = |\mathbf{r}|$]
- We know that for calculating the average probability of symbol error, we need to determine

$$\Pr(\mathbf{r} \text{ lies inside } Z_i \mid \mathbf{s}_i \text{ sent}) \equiv \Pr(z \text{ satisfies condition } C_i \mid \mathbf{s}_i \text{ sent})$$

- Hence, we need to know the statistics of z, which depends on the modulation scheme and the detection type.

Error probability …

- AWGN channel model: $\mathbf{r} = \mathbf{s}_i + \mathbf{n}$
  - Signal vector $\mathbf{s}_i = (a_{i1}, a_{i2}, \ldots, a_{iN})$ is deterministic.
  - Elements of noise vector $\mathbf{n} = (n_1, n_2, \ldots, n_N)$ are i.i.d Gaussian random variables with zero-mean and variance $N_0 / 2$. The noise vector pdf is

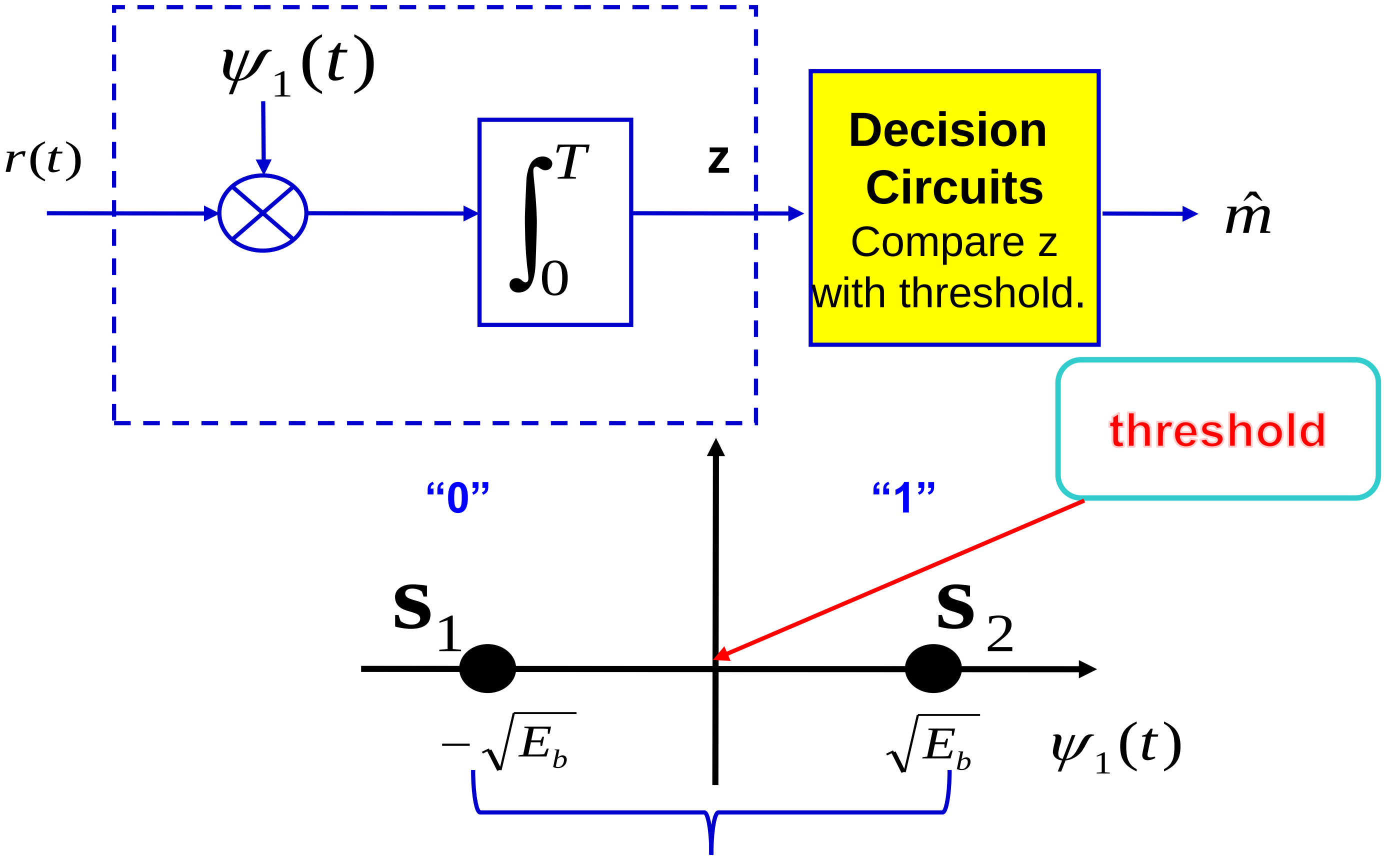
$$p_{\mathbf{n}}(\mathbf{n}) = \frac{1}{(\pi N_0)^{N/2}} \exp\left(-\frac{\|\mathbf{n}\|^2}{N_0} \right)$$

  - The elements of observed vector $\mathbf{r} = (r_1, r_2, \ldots, r_N)$ are independent Gaussian random variables. Its pdf is

$$p_{\mathbf{r}}(\mathbf{r} \mid \mathbf{s}_i) = \frac{1}{(\pi N_0)^{N/2}} \exp\left(-\frac{\|\mathbf{r} - \mathbf{s}_i\|^2}{N_0} \right)$$

Demodulation BPSK

- BPSK with *coherent* detection:



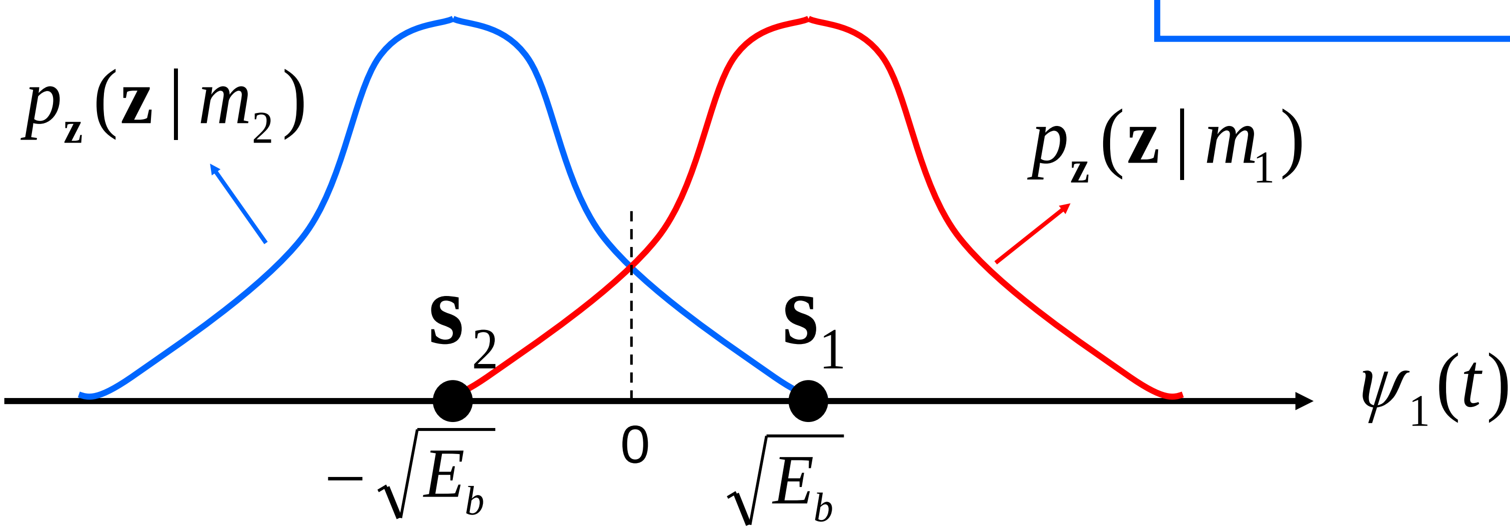
$$\|\mathbf{s}_1 - \mathbf{s}_2\| = 2\sqrt{E_b}$$

Error probability …

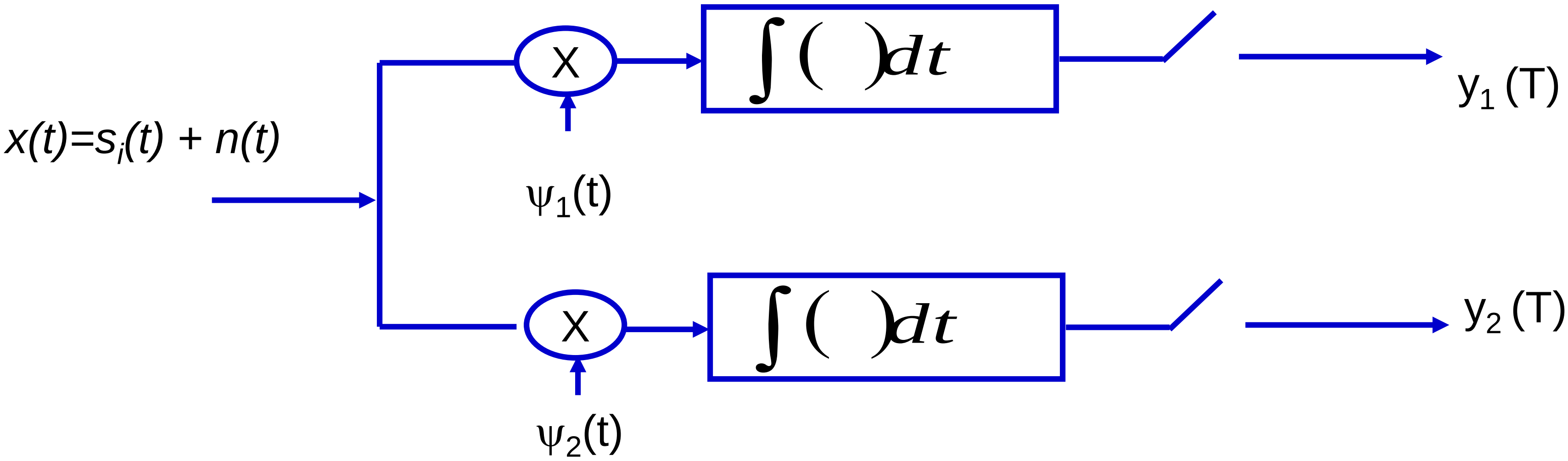
- BPSK with *coherent* detection (with perfect carrier synchronization):

$$P_B = Q\left(\frac{\|\mathbf{s}_1 - \mathbf{s}_2\|/2}{\sqrt{N_0/2}}\right)$$

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$



Coherent Detection of QPSK



$$y_1(T) = \sqrt{E_s}\cos\left[(2i-1)\frac{\pi}{4}\right] + n_1 = \pm\sqrt{\frac{E_s}{2}} + n_1$$

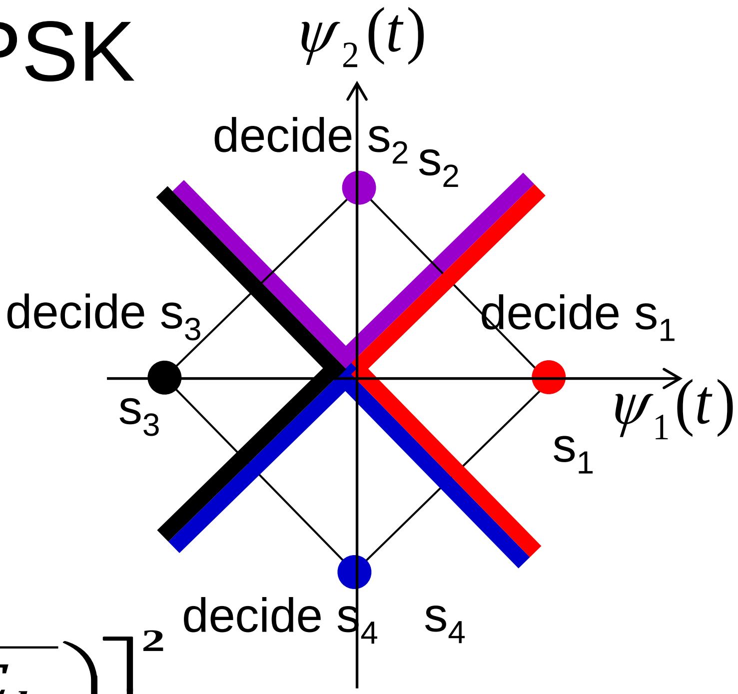
$$y_2(T) = \sqrt{E_s}\sin\left[(2i-1)\frac{\pi}{4}\right] + n_2 = \mp\sqrt{\frac{E_s}{2}} + n_2$$

QPSK can be seen as two binary PSK acting independently.

Demodulation M-PSK

- Coherent detection of Q-PSK

Decision Region QPSK



$$p_C = (1 - p_{BPSK-I})^2 = \left[1 - Q\left(\sqrt{\frac{2E_b}{N_o}}\right)\right]^2$$

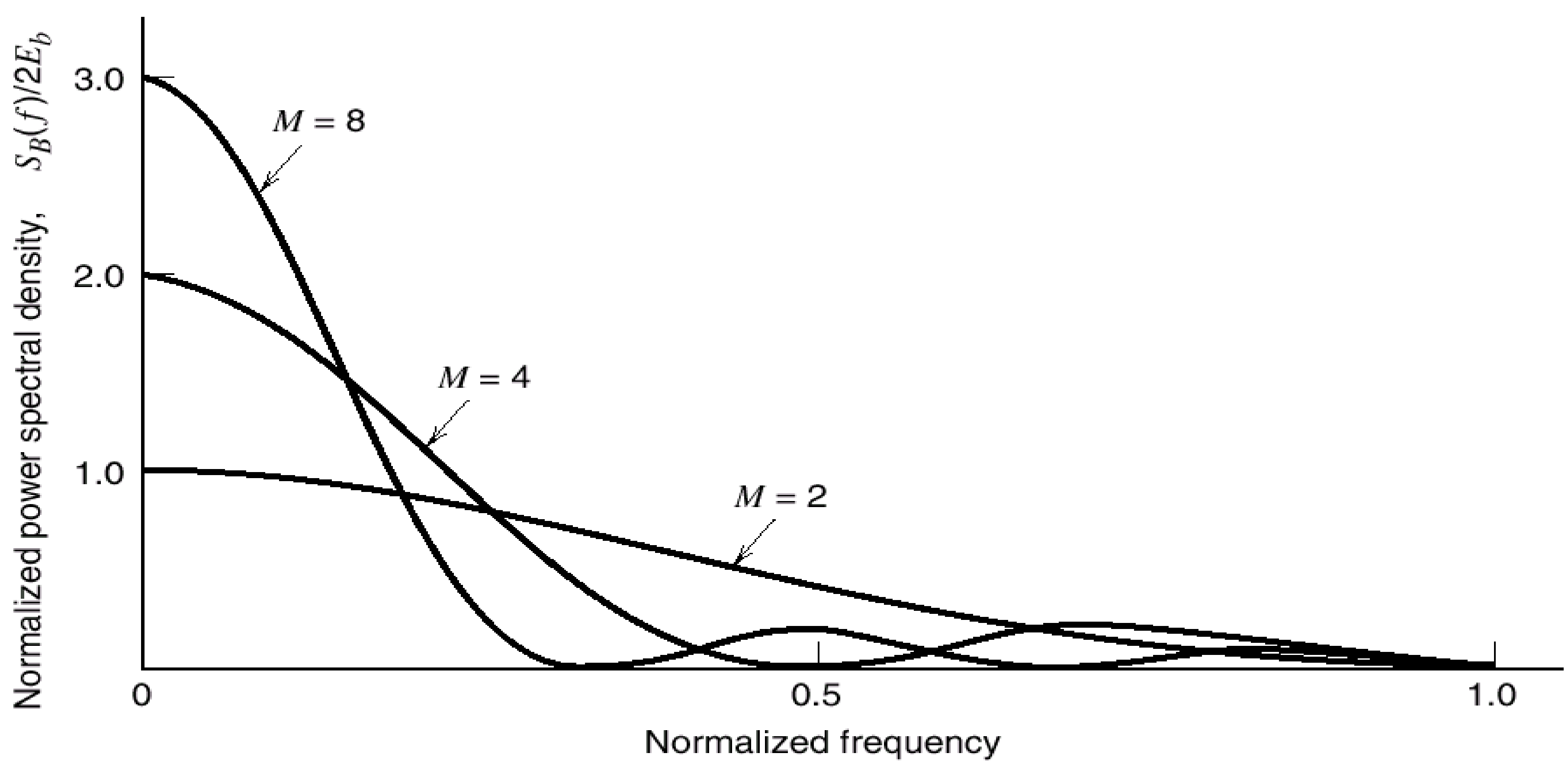
$$p_e = 1 - p_c = 2Q\left(\sqrt{\frac{2E_b}{N_o}}\right)\left[1 - \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_o}}\right)\right]$$

$$p_e \approx Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Power Spectra of M-Ary PSK

$$S_B(f) = 2E \operatorname{sinc}^2(Tf)$$

$$S_B(f) = 2E_b \log M \operatorname{sinc}^2(T_b f \log_2 M)$$



QPSK vs. BPSK

- Let's compare the two based on BER and bandwidth

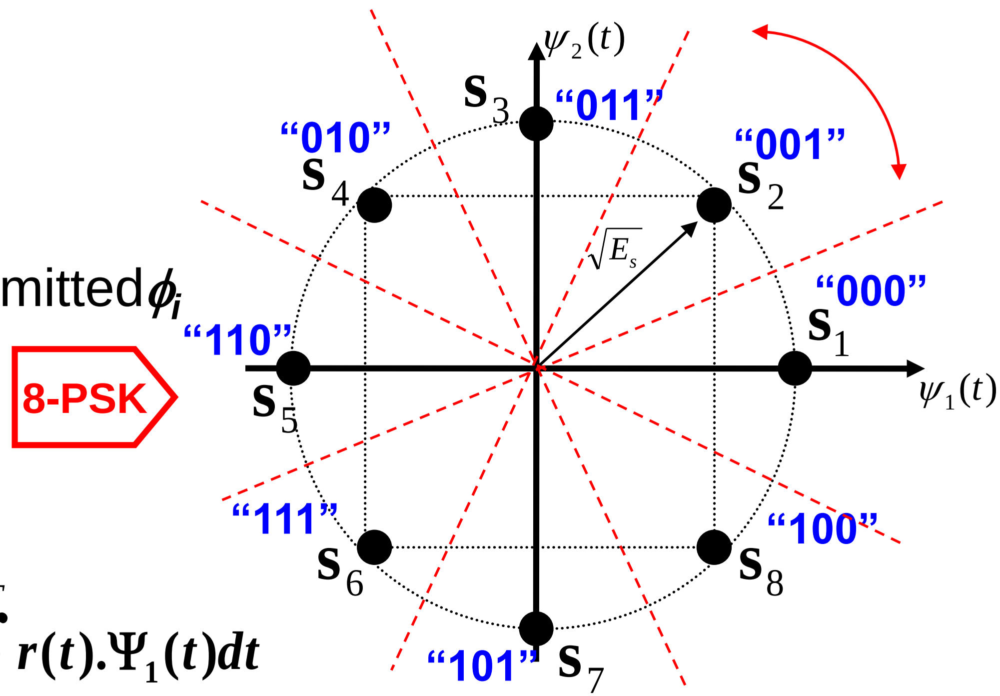
BER		Bandwidth	
BPSK	QPSK	BPSK	QPSK
$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	R_b	$R_b/2$

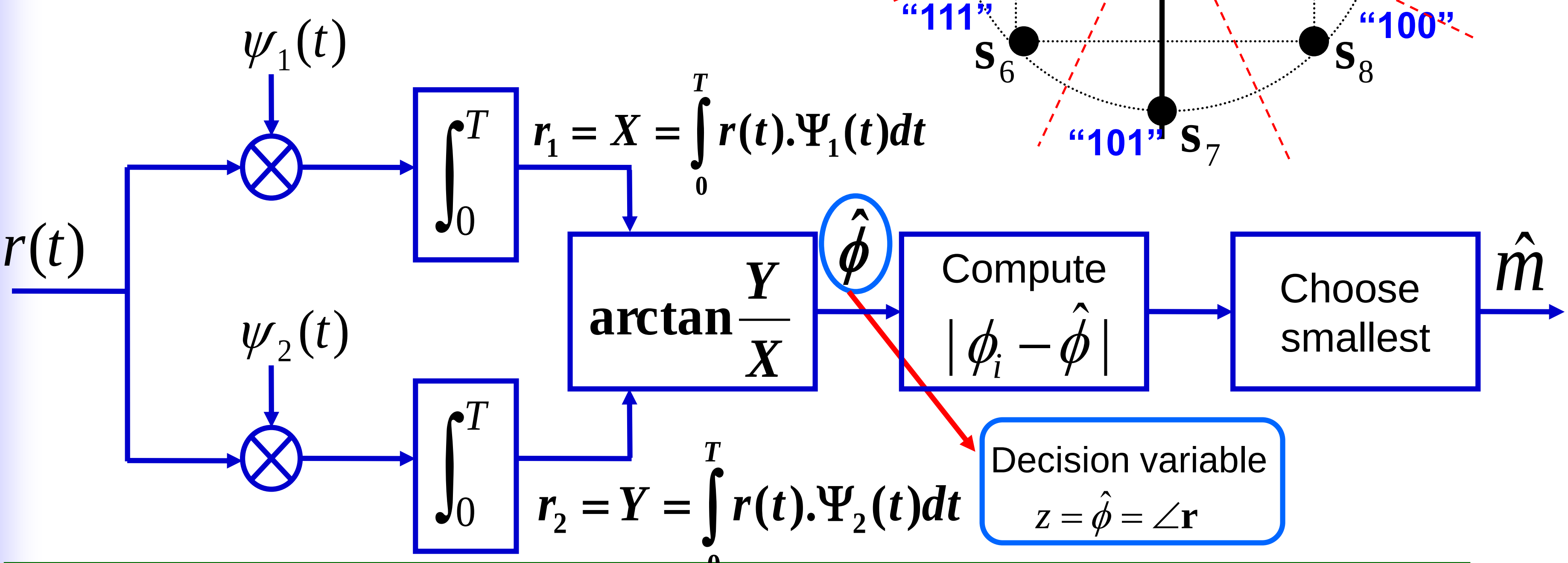
EQUAL

Error probability …

- ***Coherent*** **detection of M-PSK**

$\hat{\phi}$ is a noisy estimate of the transmitted ϕ_i





Error probability …

Coherent detection of MPSK …

- The detector compares the phase of observation vector to M-1 thresholds.
- Due to the circular symmetry of the signal space, we have:

$$P_E(M) = 1 - P_C(M) = 1 - \frac{1}{M}\sum_{m=1}^{M} P_c(\mathbf{s}_m) = 1 - P_c(\mathbf{s}_1) = 1 - \int_{-\pi/M}^{\pi/M} p_{\hat{\phi}}(\phi)\, d\phi$$

where

$$p_{\hat{\phi}}(\phi) \approx \sqrt{\frac{2}{\pi}\frac{E_s}{N_0}}\cos(\phi)\exp\left(-\frac{E_s}{N_0}\sin^2\phi\right); \quad |\phi| \le \frac{\pi}{2}$$

- It can be shown that (**for M > 4**)

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$ or $$\boldsymbol{P_E(M) \approx 2Q\left(\sqrt{\frac{2(\log_2 M)E_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)}$$

Non-coherent detection

- Non-coherent detection:
  - No need in a reference in phase with the received carrier
  - Less complexity as compared to coherent detection at the price of higher error rate.

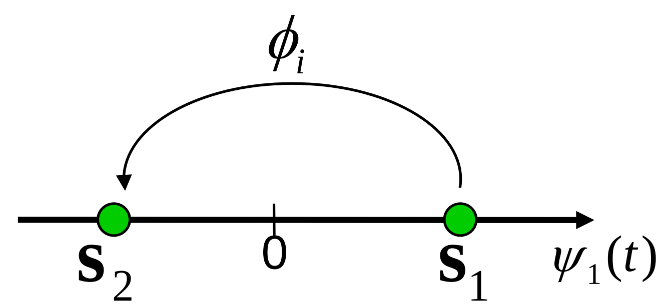
Differential PSK…

- Differential encoding of the message
  - The symbol phase changes if the current bit is different from the previous bit.

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left(\omega_0 t + \theta_i(t)\right), \quad 0 \le t \le T, \quad i = 1, \ldots, M$$

$$\boldsymbol{c(k) = \overline{c(k-1) \oplus m(k)} = c(k-1) \otimes m(k)}$$

Symbol index: k	0	1	2	3	4	5	6	7
Data bits: $\boldsymbol{m(k)}$		1	1	0	1	0	1	1
Diff. encoded bits: $\boldsymbol{c(k)}$	1	1	1	0	0	1	1	1
Symbol phase: θ_k	π	π	π	0	0	π	π	π



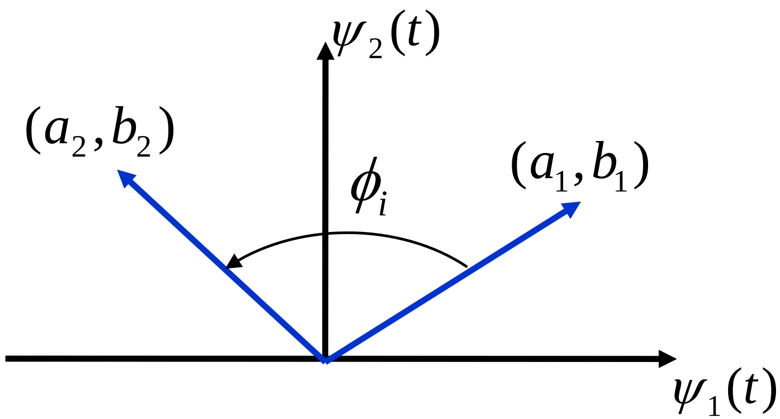
$$\theta_k(nT) = \theta_k((n-1)T) + \phi_i(nT)$$

Coherent detection for diff encoded mod.

- assumes slow variation in carrier-phase mismatch during two symbol intervals.
- correlates the received signal with basis functions
- uses the phase difference between the current received vector and previously estimated symbol

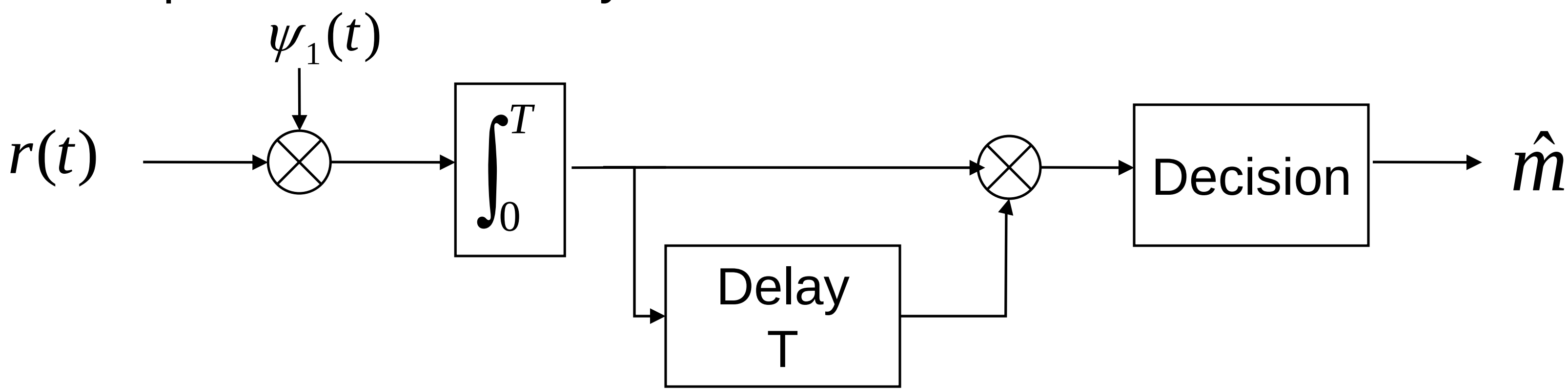
$$r(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_0 t + \theta_i(t) + \alpha\right) + n(t), \quad 0 \le t \le T$$

$$\left(\theta_i(nT) + \alpha\right) - \left(\theta_j((n-1)T) + \alpha\right) = \theta_i(nT) - \theta_j((n-1)T) = \phi_i(nT)$$

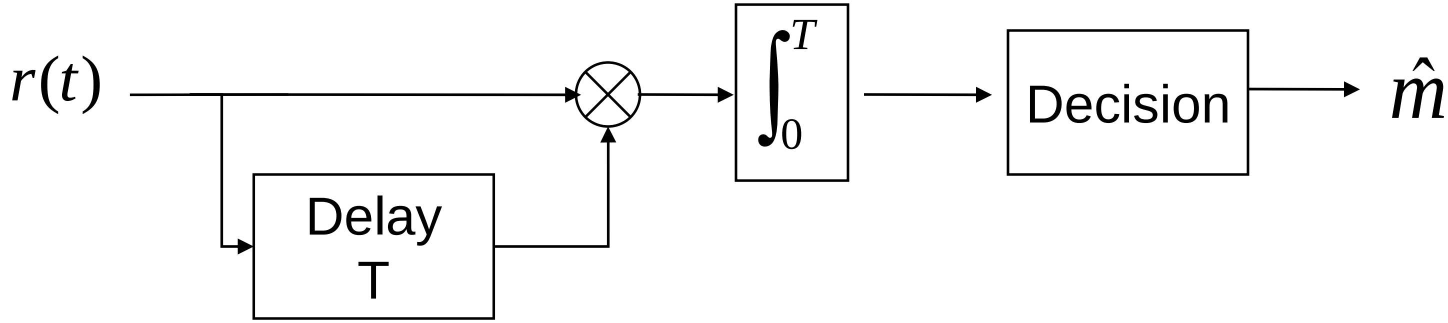


DPSK detection …

- Optimum differentially coherent detector



- Sub-optimum differentially coherent detector



  - Performance degradation about 3 dB by using sub-optimum detector

Error probability for DPSK detection …

The symbol error performance for differentially coherent detector of M-ary DPSK (for large Es/No) :

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2(\log_2 M)E_b}{N_0}}\sin\left(\frac{\pi}{\sqrt{2M}}\right)\right)$$

Non-Coherent Detection of Binary PSK-Differential PSK (DPSK)

The transmitted signal:

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_i) \qquad i = 0,1$$

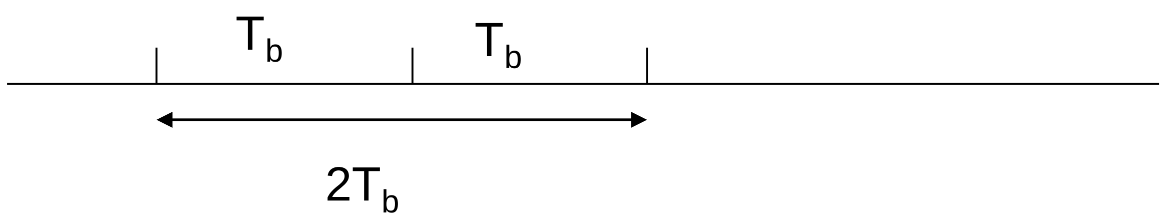
$$0 \le t \le T_b$$

The received signal:

α is assumed to change slowly reletive to consecutive symbols

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_i + \alpha) + n(t) \qquad i = 0,1$$

$$0 \le t \le T_b$$



$$[\theta_i(T_2) + \alpha] - [\theta_i(T_1) + \alpha] = 0$$ ← Equal phases

$$[\theta_i(T_2) + \alpha] - [\theta_j(T_1) + \alpha] = \pi$$ ← different phases

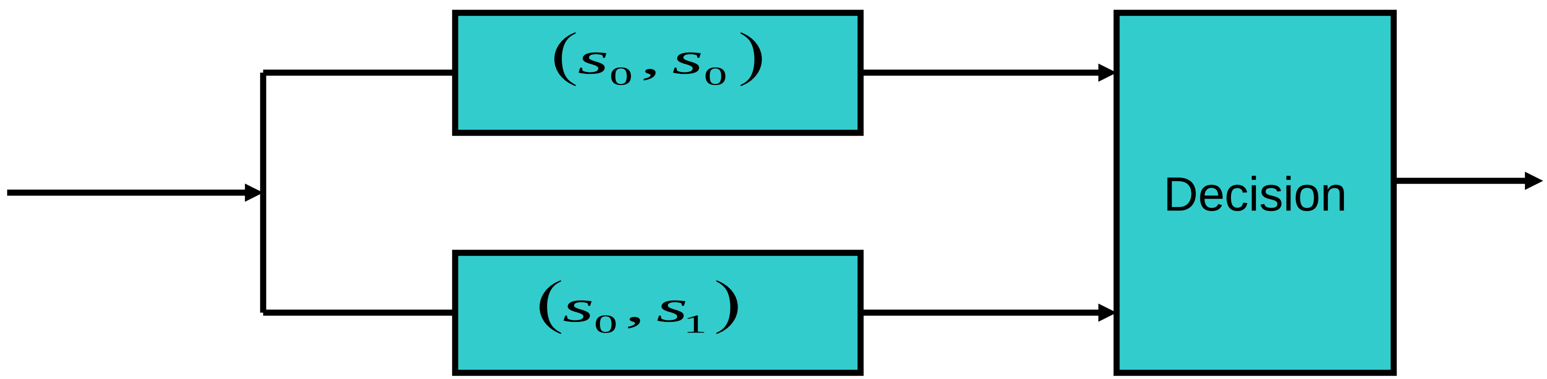
Non-Coherent Differential PSK (DPSK)

Effectively in DPSK signaling we are transmitting each bit with the binary signaling pair:

$$(s_0, s_1)$$
$$(s_1, s_0)$$
$$(s_1, s_1)$$
$$(s_0, s_0)$$

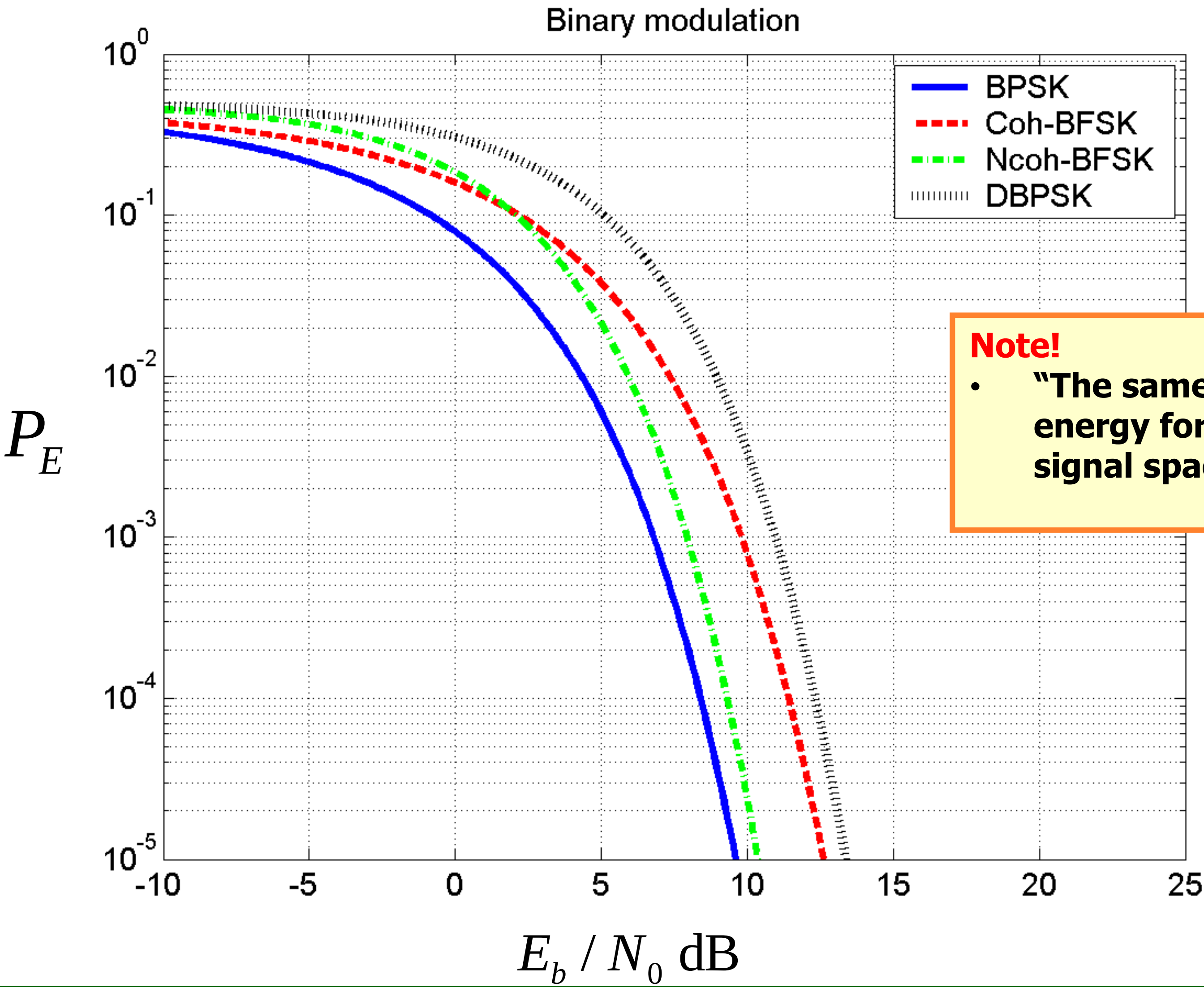
$$0 \le t \le 2T_b$$

Filters matched to signal envelope



$$p_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

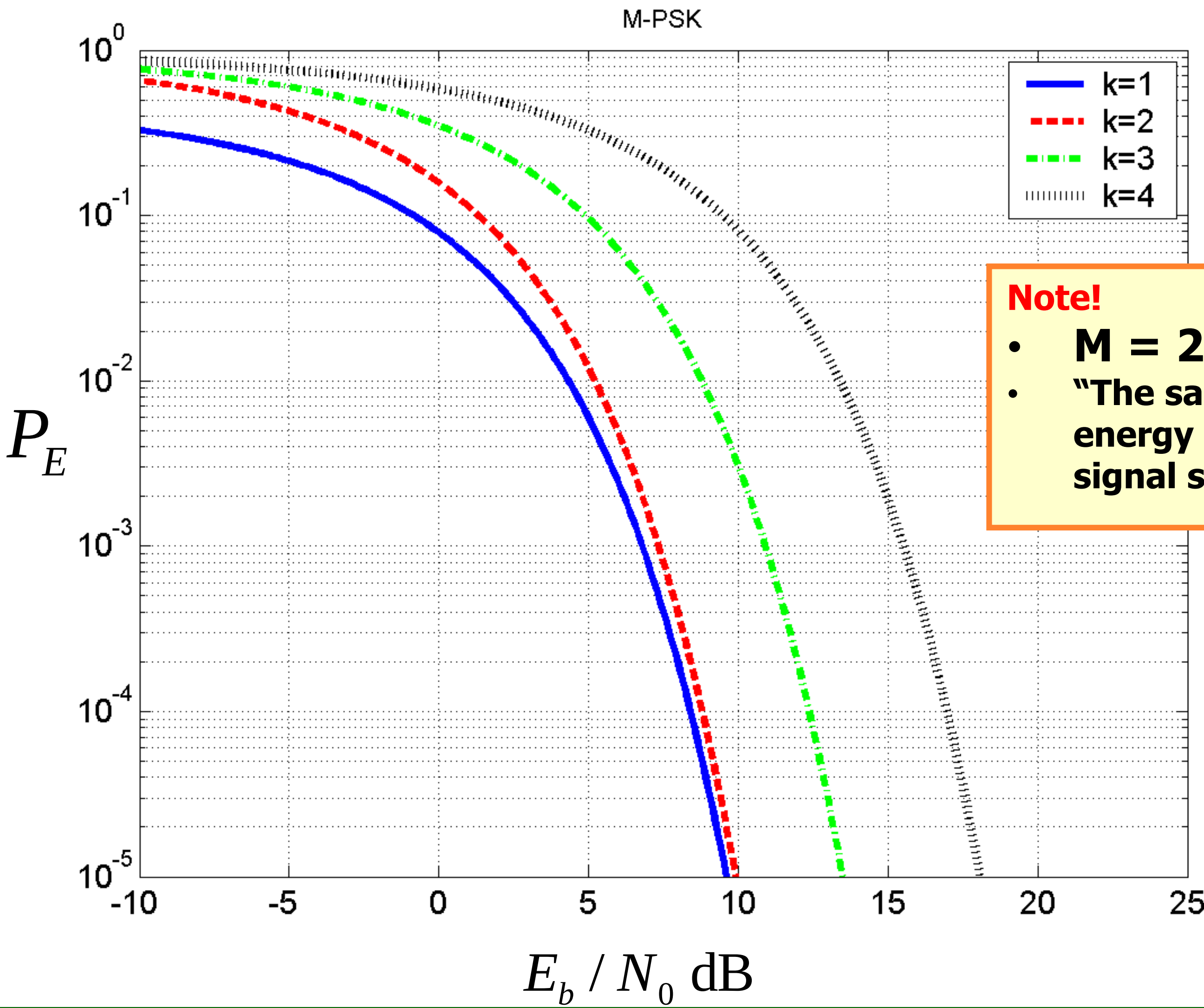
Probability of symbol error for binary modulation



Note!

- **"The same average symbol energy for different sizes of signal space"**

Probability of symbol error for M-PSK



Note!

- **M = 2^k**
- **"The same average symbol energy for different sizes of signal space"**

Example of samples of matched filter output for some bandpass modulation schemes