Sistem Komunikasi 1

Bab 2

Transformasi Fourier

Analog Vs. Digital (1)

Advantages of Digital Communication

As the signals are digitized, there are many advantages of digital communication over analog communication, such as:

- The effect of distortion, noise, and interference is much less in digital signals as they are less affected.
- Digital circuits are more reliable.
- Digital circuits are easy to design and cheaper than analog circuits.
- The hardware implementation in digital circuits, is more flexible than analog.

Analog Vs. Digital (2)

- The occurrence of cross-talk is very rare in digital communication.
- The signal is un-altered as the pulse needs a high disturbance to alter its properties, which is very difficult.
- Signal processing functions such as encryption and compression are employed in digital circuits to maintain the secrecy of the information.
- The probability of error occurrence is reduced by employing error detecting and error correcting codes.
- Spread spectrum technique is used to avoid signal jamming.

Analog Vs. Digital (3)

- Combining digital signals using Time Division Multiplexing (TDM) is easier than combining analog signals using Frequency Division Multiplexing (FDM).
- The configuring process of digital signals is easier than analog signals.
- Digital signals can be saved and retrieved more conveniently than analog signals.
- Many of the digital circuits have almost common encoding techniques and hence similar devices can be used for a number of purposes.
- The capacity of the channel is effectively utilized by digital signals.

Formula Transformasi Fourier

$$S(f) = \int_{-\infty}^{+\infty} s(t).e^{-j2\pi ft}\,dt$$

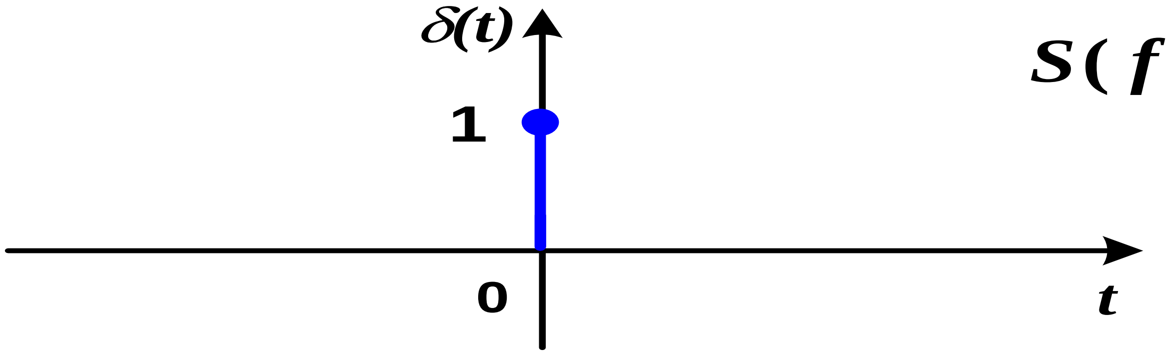
- *S(f)* dinamakan Transformasi Fourier dari s(t)

- Jika Transformasi Fourier *S(f)* suatu sinyal diketahui maka kita dapat menghitung persamaan sinyal dalam domain waktu s(t) dengan formula Inverse Transformasi Fourier

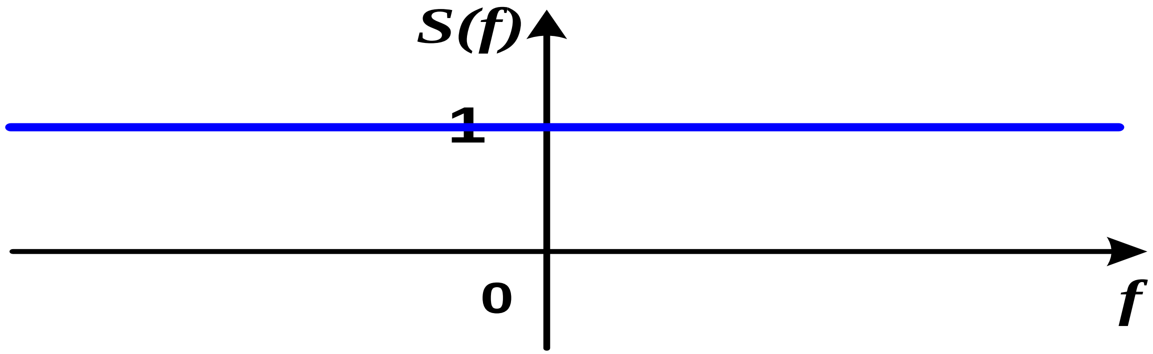
$$s(t) = \int_{-\infty}^{+\infty} S(f).e^{j2\pi ft}\,df$$

Beberapa Transformasi penting

- Transformasi Fourier impulse (sinyal delta dirac):

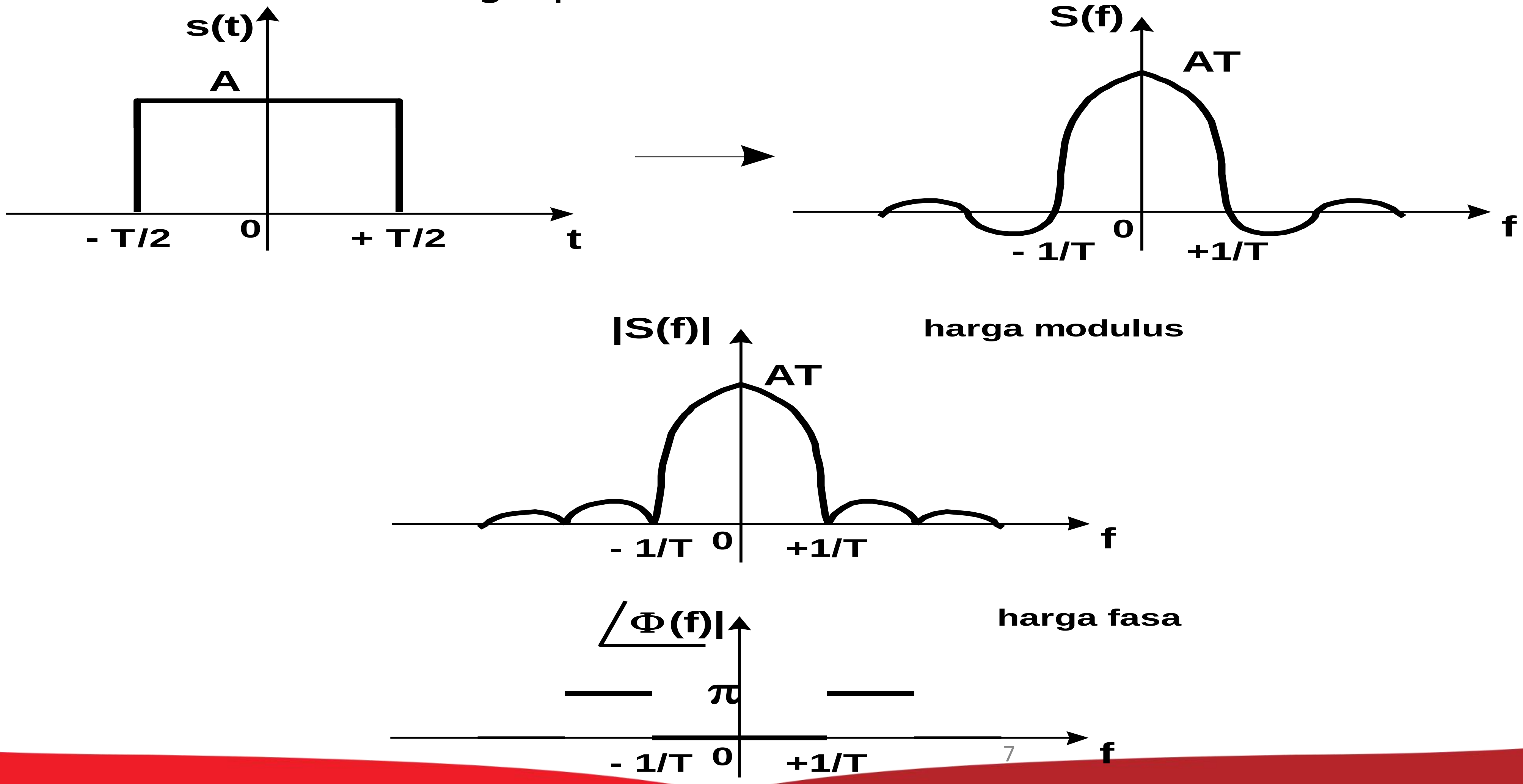


$$S(f) = \int_{-\infty}^{+\infty} \delta(t).e^{-j2\pi ft}\,dt = 1$$



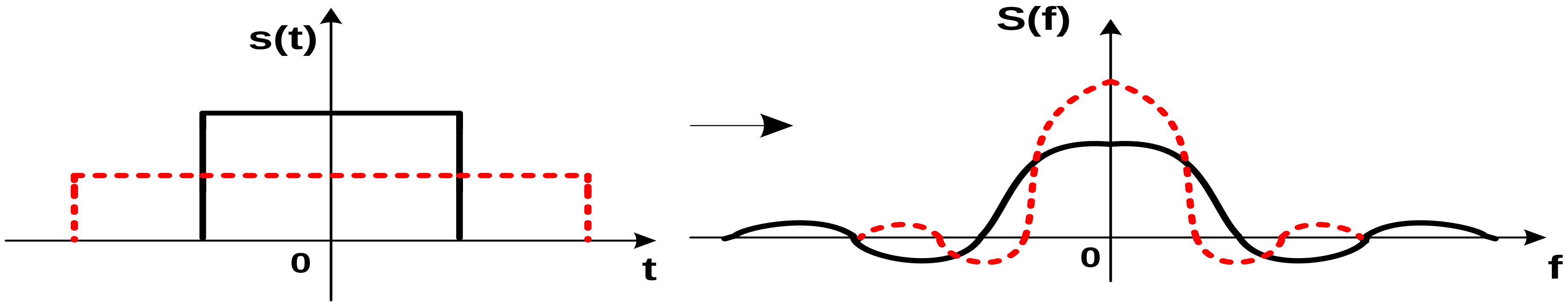
Beberapa Transformasi penting

- Transformasi Fourier dari fungsi pulsa:



Sifat-sifat Transformasi Fourier (yang sering dipakai di siskom)

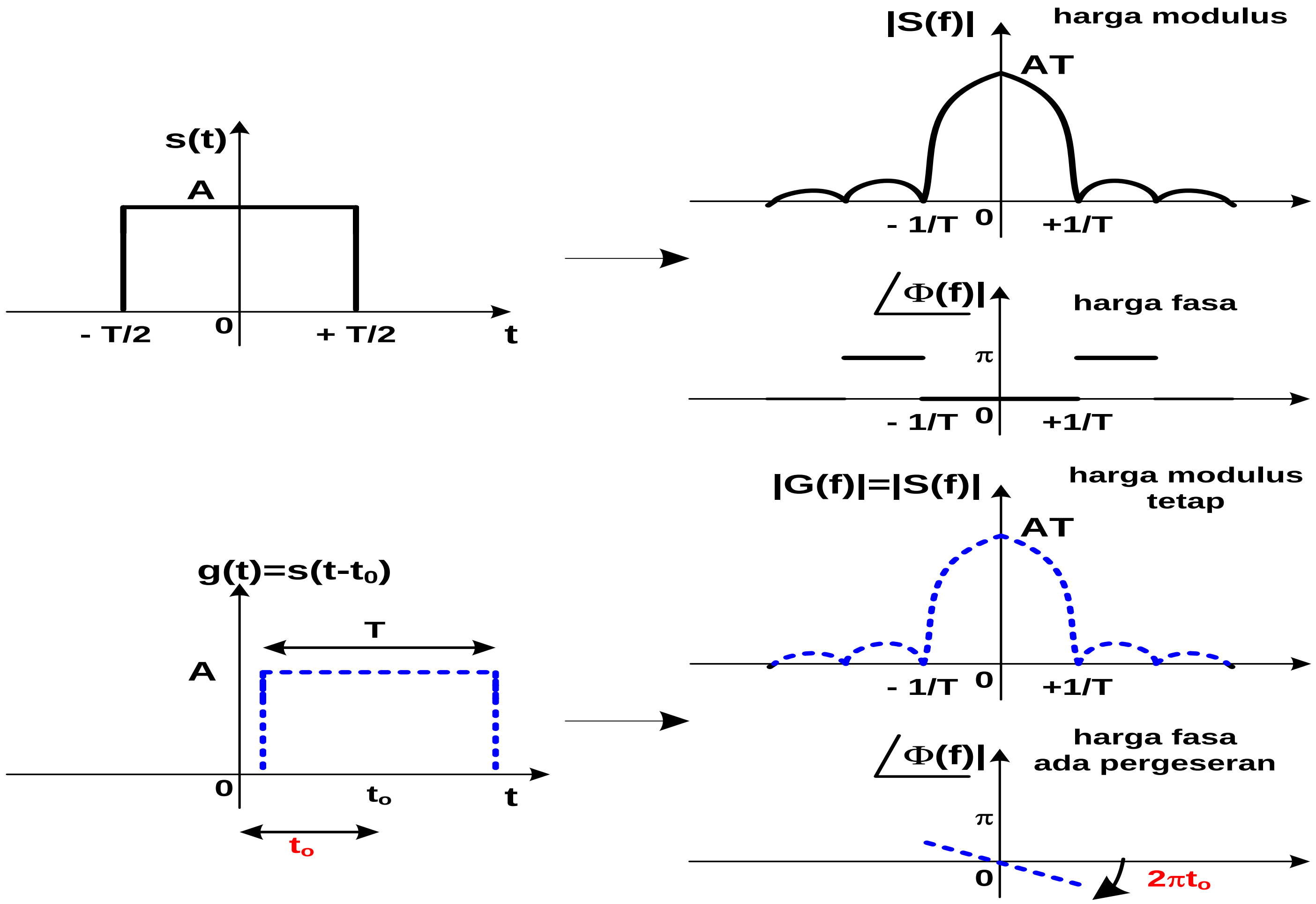
a. Time Scaling



Sifat-sifat Transformasi Fourier

b. Time shifting

Bila $s(t) \leftrightarrow S(f)$ maka $s(t-t_o) \leftrightarrow S(f).e^{-j2\pi f t_o}$



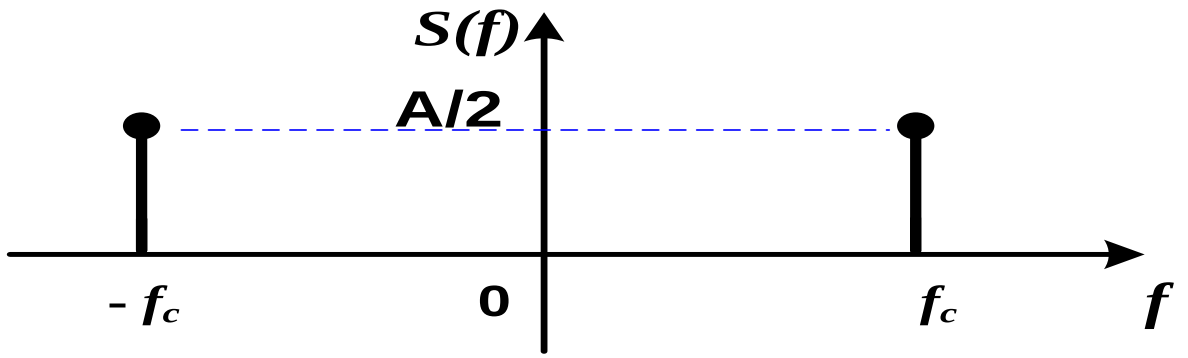
Sifat-sifat Transformasi Fourier

c. Frequency shifting

Bila $s(t) \leftrightarrow S(f)$ maka $S(f-f_o) \leftrightarrow s(t).e^{-j2\pi f_o t}$

- Contoh : $s(t) = A \cos 2\pi f_c t = \frac{A}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$

- maka $S(f) = \frac{A}{2}\delta(f + f_c) + \frac{A}{2}\delta(f - f_c)$



d. Transformasi Fourier Sinyal Periodik

Bila ***x(t)*** $\leftrightarrow$ ***X(f)*** (untuk sinyal tidak periodik)

Maka untuk

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

(➔x(t) periodik dengan periode T_o)

Transformasi fourier dari $x_p(t)$

$$X_p(f) = \frac{1}{T_0} \sum_{m=-\infty}^{+\infty} X\left(\frac{m.f}{T_o}\right).\delta\left(f - \frac{m}{T_0}\right)$$

Sifat-sifat Transformasi Fourier

e. Integrasi pada kawasan waktu:

Bila ***s(t) ↔ S(f),*** kemudian menghasilkan ***S(0)=0,*** ***maka :***

$$\int_{-\infty}^{t} s(t).dt \Leftrightarrow \frac{1}{j2\pi f}.S(f)$$

f. Diferensiasi pada kawasan waktu:

Bila ***s(t) ↔ S(f),*** jika pada kawasan waktu dilakukan diferensiasi sekali, ***maka :***

$$\frac{d}{dt}s(t) \Leftrightarrow j2\pi f.S(f)$$

g. Konvolusi pada kawasan waktu:

Bila $s_1(t) \leftrightarrow S_1(f)$ dan $s_2(t) \leftrightarrow S_2(f)$,

maka :

$$\int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau)d\tau \Leftrightarrow S_1(f).S_2(f)$$

h. Perkalian pada kawasan waktu:

Bila $s_1(t) \leftrightarrow S_1(f)$ dan $s_2(t) \leftrightarrow S_2(f)$,

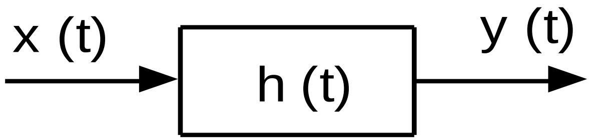
maka :

$$s_1(t).s_2(t) \Leftrightarrow \int_{-\infty}^{\infty} S_1(\lambda).S_2(f-\lambda)d\lambda$$

Transmisi Sinyal melalui Sistem Linier

Respon Time :

Time Domain



h (t) ≡ respon impuls

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

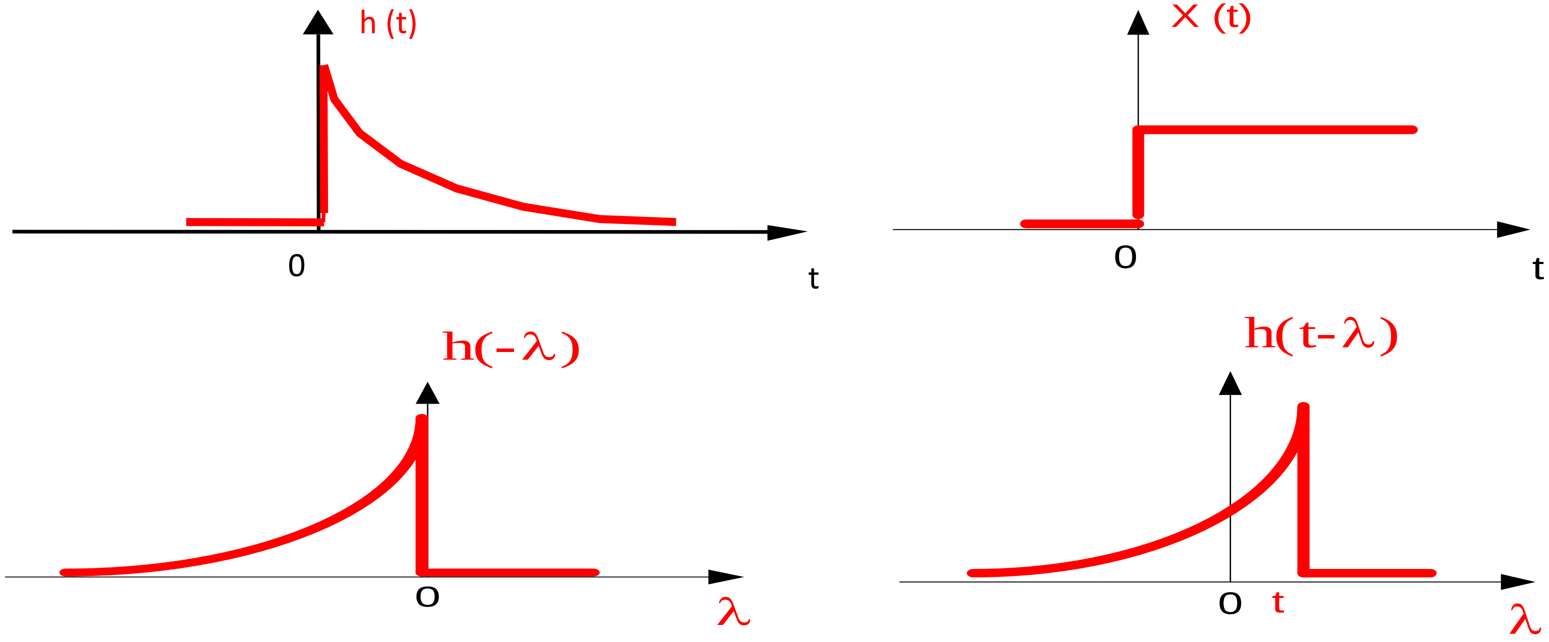
$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

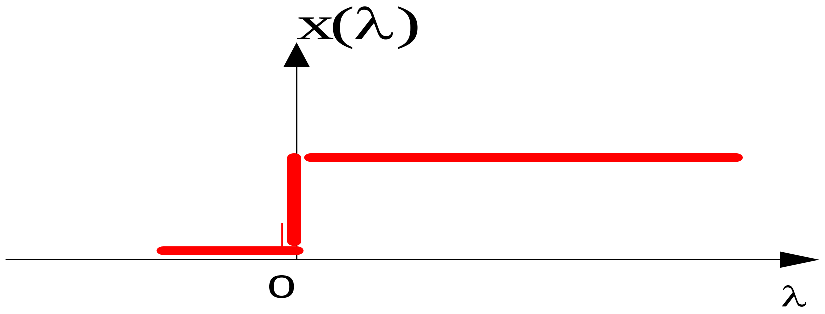
$$= x(t) \otimes h(t)$$

$$= h(t) \otimes x(t)$$

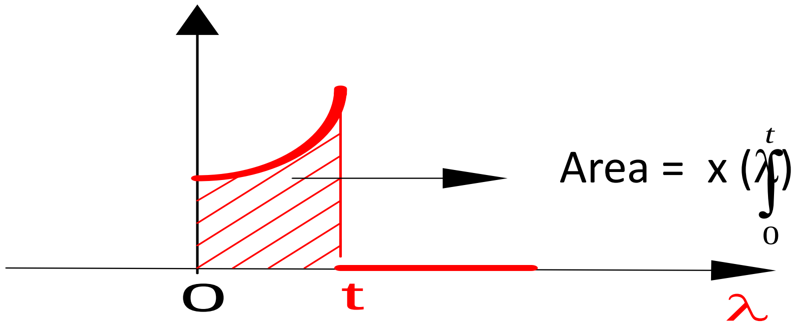
Perhitungan Konvolusi :

Representasi Grafis ; contoh

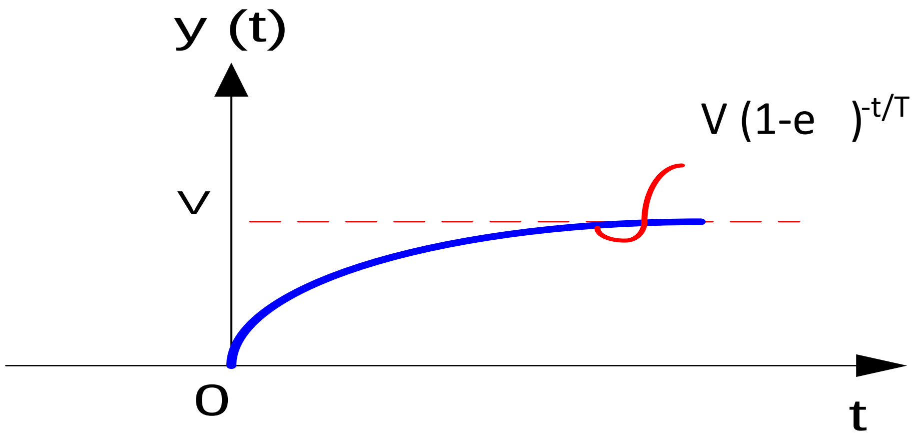




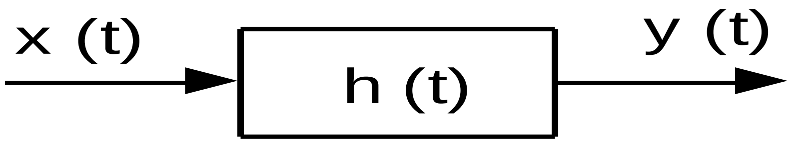
x(λ) h(t−λ)

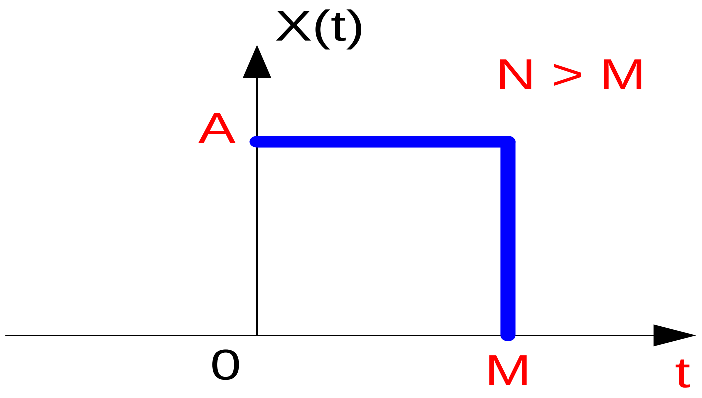


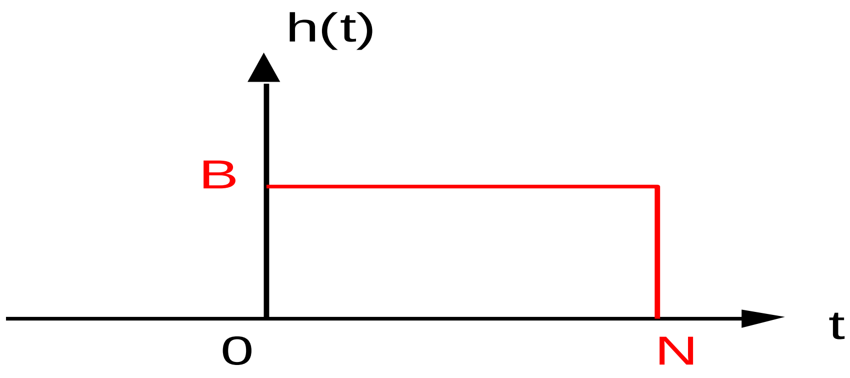
Area = $\int_0^t x(\lambda)\, h(t-\lambda)\, d\lambda$

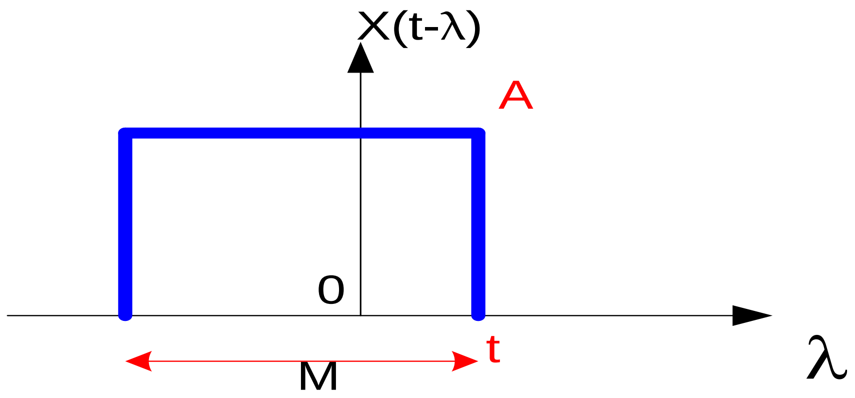


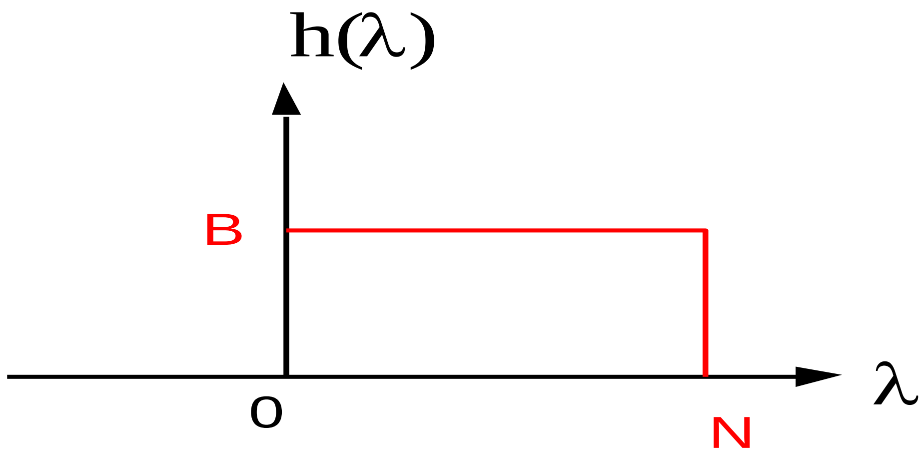
Contoh Perhitungan Konvolusi dgn representasi Grafis :

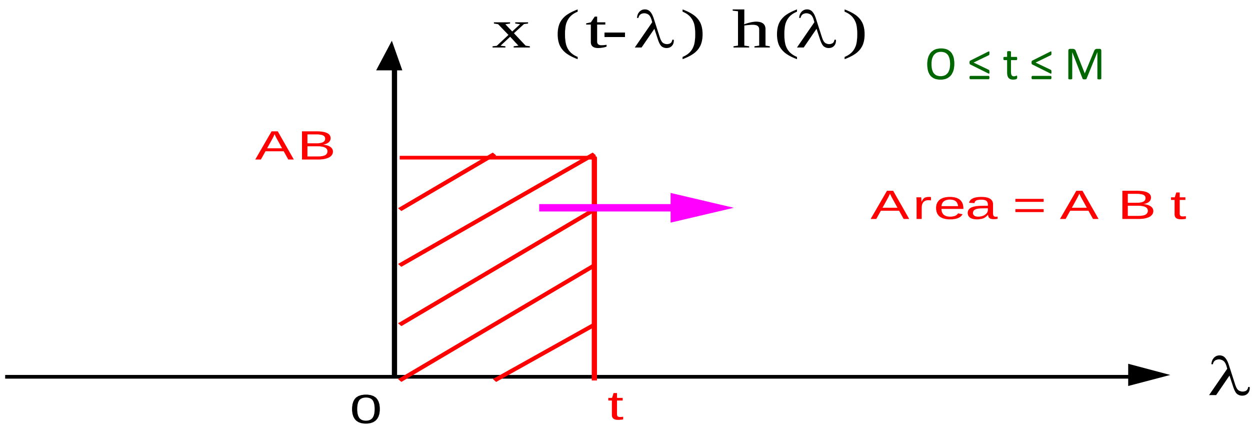










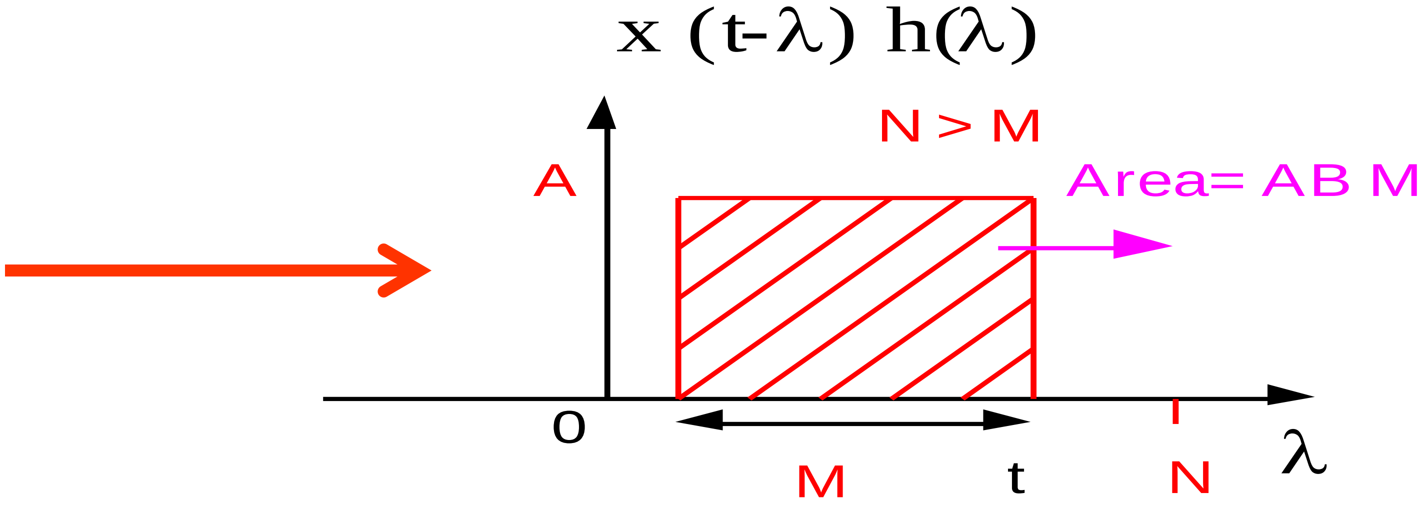


Perhitungan

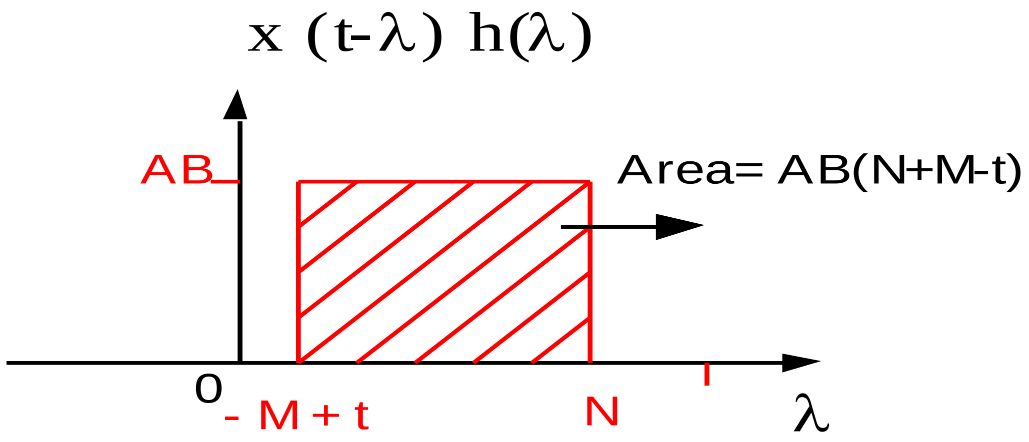
Karena N > M :

untuk 0 ≤ t ≤ M : y(t) = ABt

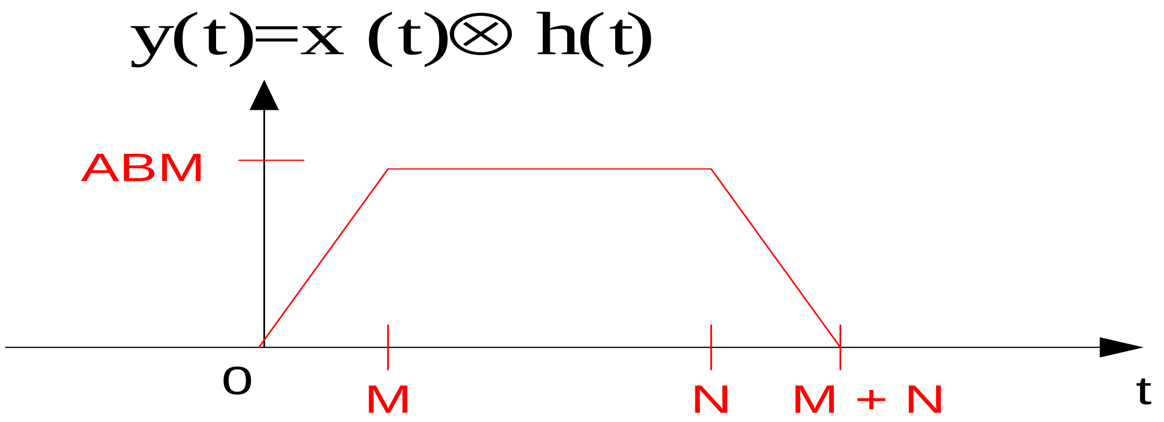
untuk M ≤ t ≤ N :



untuk t ≥ N :



Sehingga:

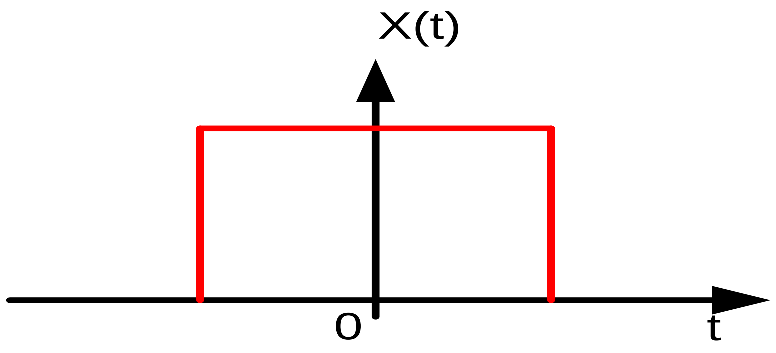


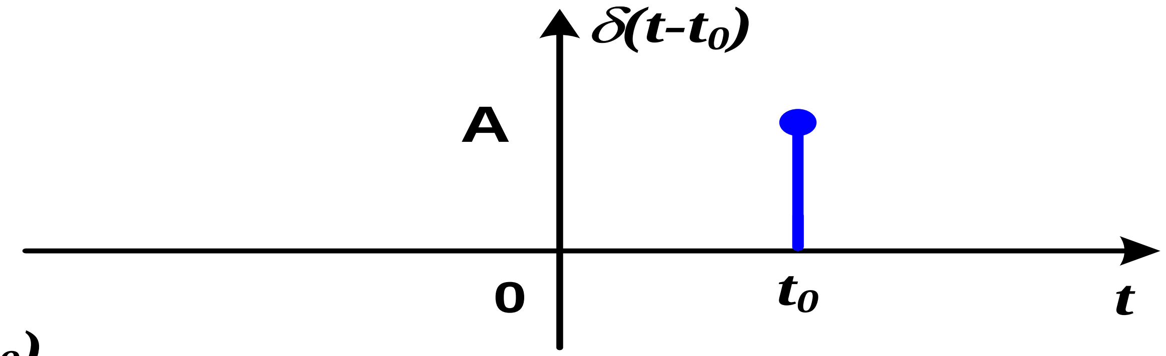
Kasus Khusus :

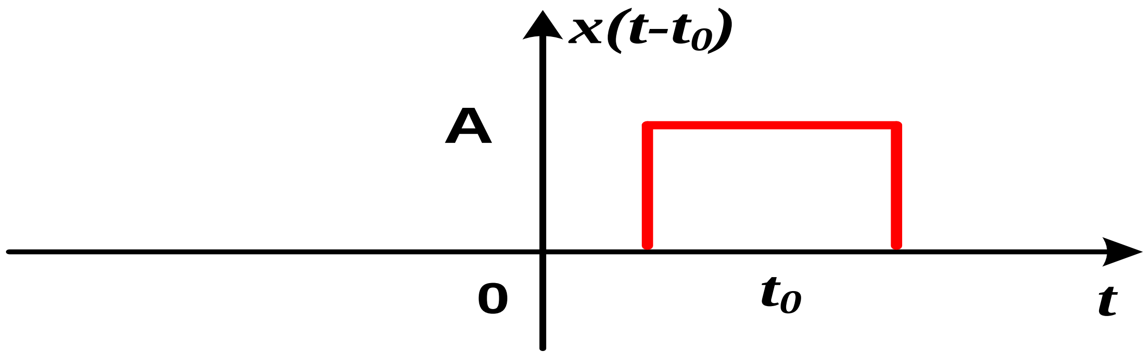
Konvolusi dengan fungsi δ (t - to)

- $x(t) \otimes \delta(t - to) = x(t - \lambda) \int_{-\infty}^{\infty} \delta(\lambda - to)\, d\lambda = x(t - to)$

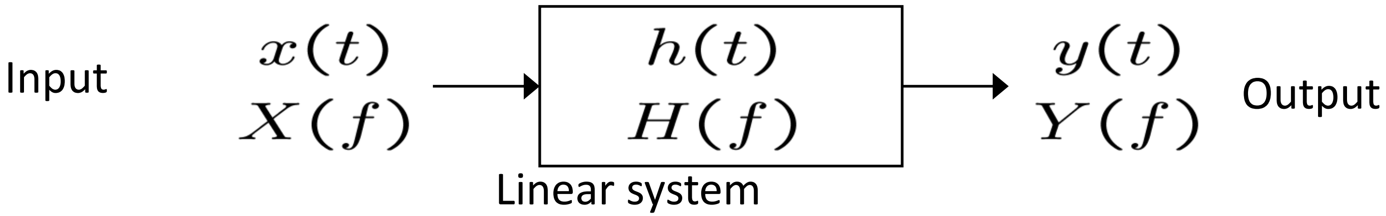
- $x(t) \otimes A\,\delta(t - to) = A\,x(t - to)$







Transmisi Sinyal Melalui Sistem Linier



- Deterministic signals:

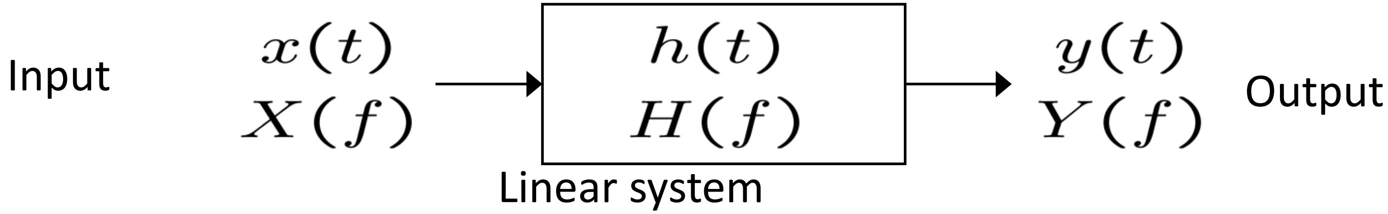
$$Y(f) = X(f)H(f)$$

- Random signals:

$$G_Y(f) = G_X(f)|H(f)|^2$$

- Y(f) = Sinyal output dalam domain frekuensi
- X(f) = Sinyal input dalam domain frekuensi
- H(f) = Respons frekuensi sistem linier
- G_Y(f) = PSD (Power Spectral Density) sinyal output
- G_X(f) = PSD (Power Spectral Density) sinyal input

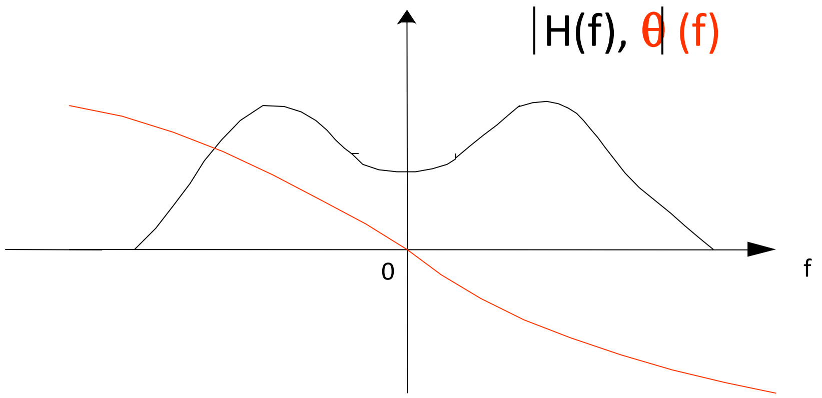
Sistem Lowpass vs Bandpass



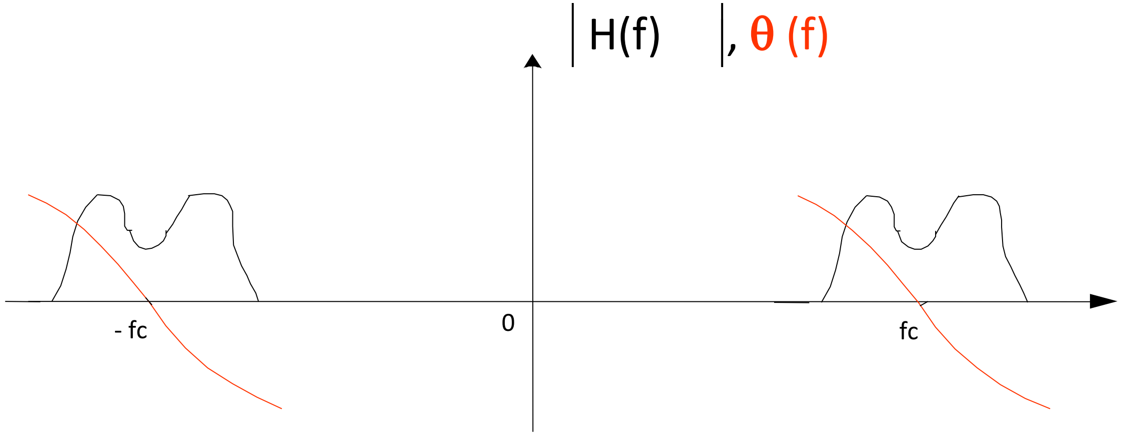
Jika h (t) riil $\Rightarrow$ H (f) kompleks $\rightarrow$ | H (f) | merupakan fungsi genap

$\rightarrow$ θ (f) merupakan fungsi ganjil

Sistem “lowpass”



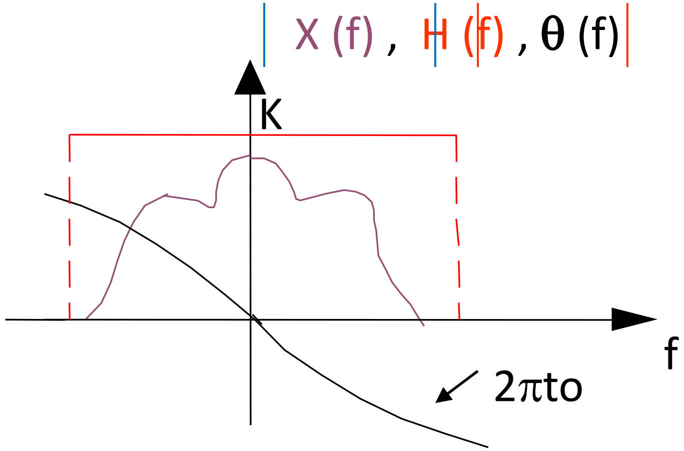
Sistem “bandpass”



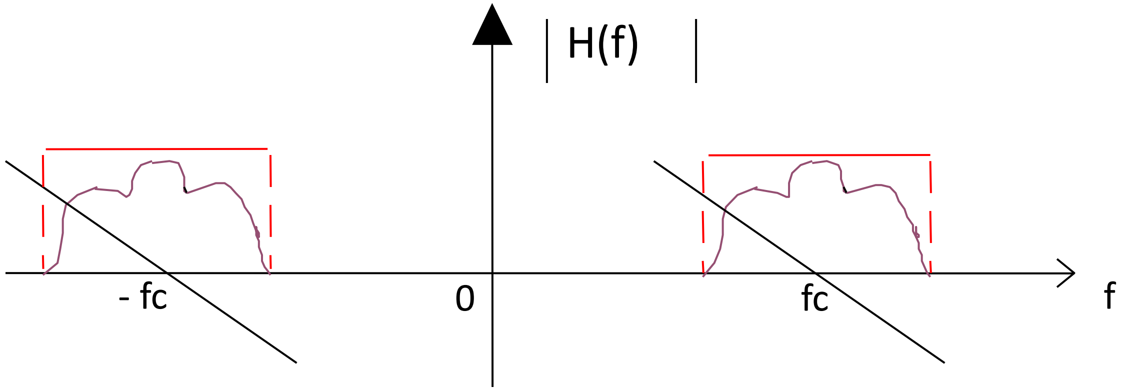
• Kondisi "distortionless transmission"

$y(t) = K.X(t - to)$

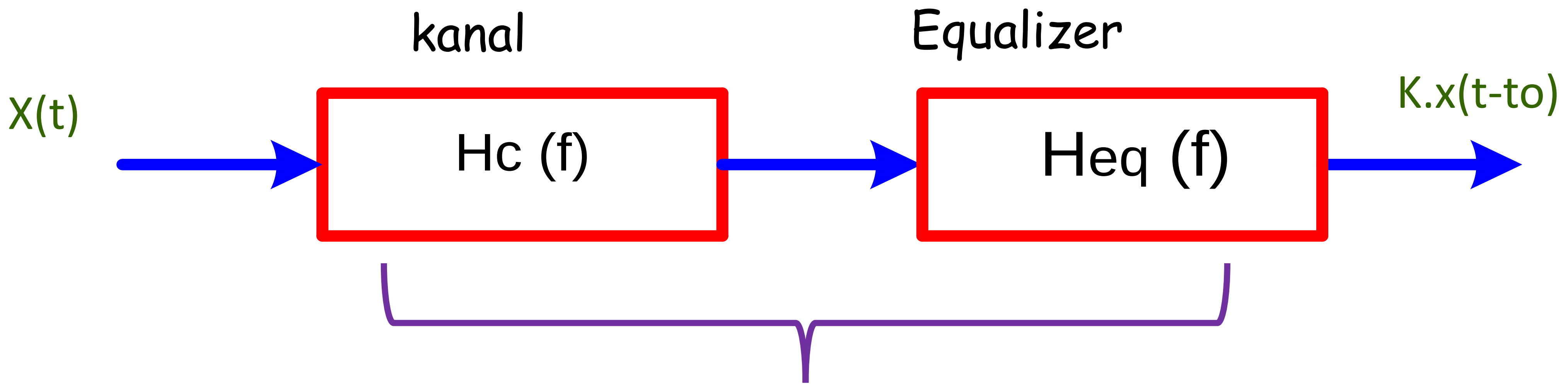
$H(f) = K\, e^{-j2\pi fto}$



• Untuk sistem "bandpass"



• Distorsi Linier dan Prinsip Ekualisasi Kanal

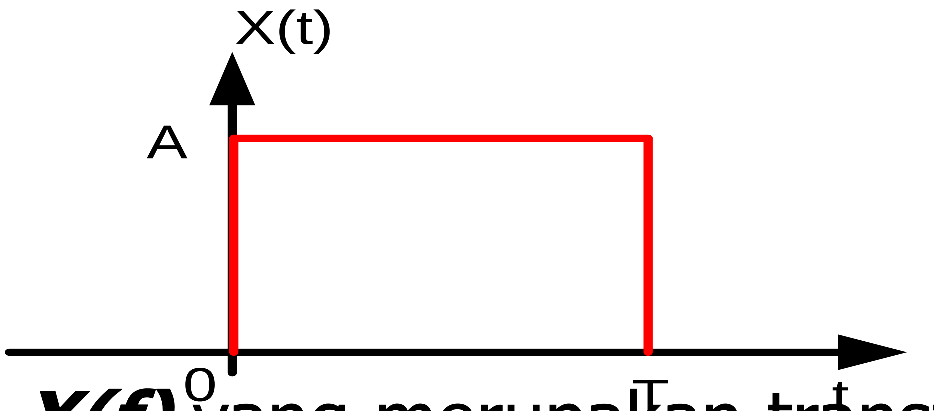


$$H_c(f)\, H_{eq}(f) = K\, e^{-j2\pi f t_o}$$

$$H_{eq}(f) = \frac{K\, e^{-j2\pi f t_o}}{H_c(f)}$$

Latihan Soal

1. Perhatian gambar sinyal ***x(t)*** diawah ini :



a. Tentukan ***X(f)*** yang merupakan transformasi fourier dari sinyal tersebut !

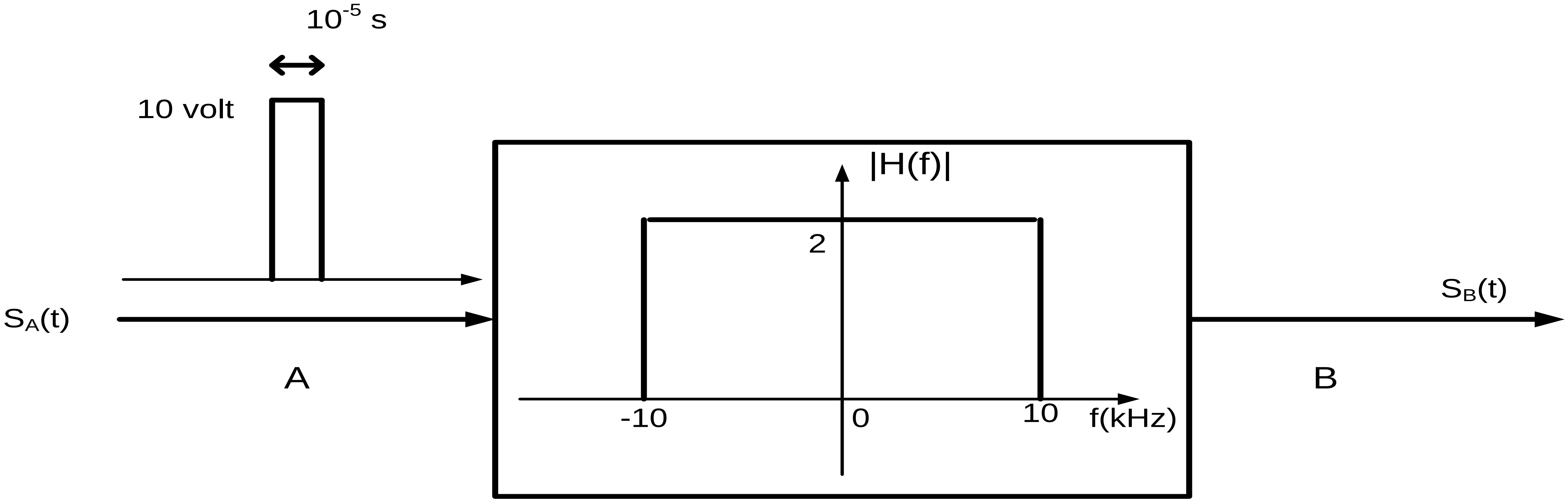
b. Jika sinyal ***z(t)= x(t).y(t)***

   dimana ***y(t) = Cos (4π t/T)***, tentukan ***Z(f)*** !

c. Gambarkan z(t) dan Z(f)

Latihan Soal

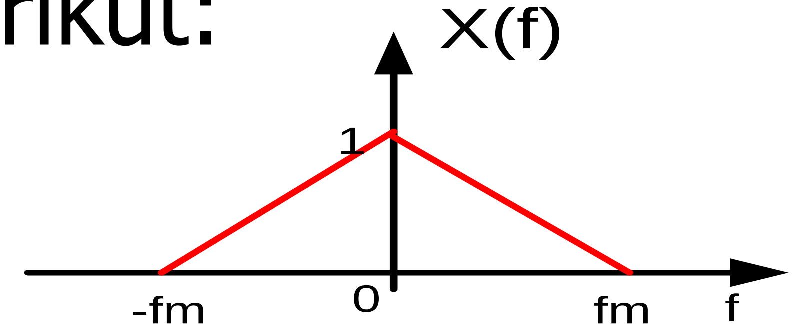
2. Suatu sinyal memasuki sistem yang diwakili oleh LPF berikut ini :

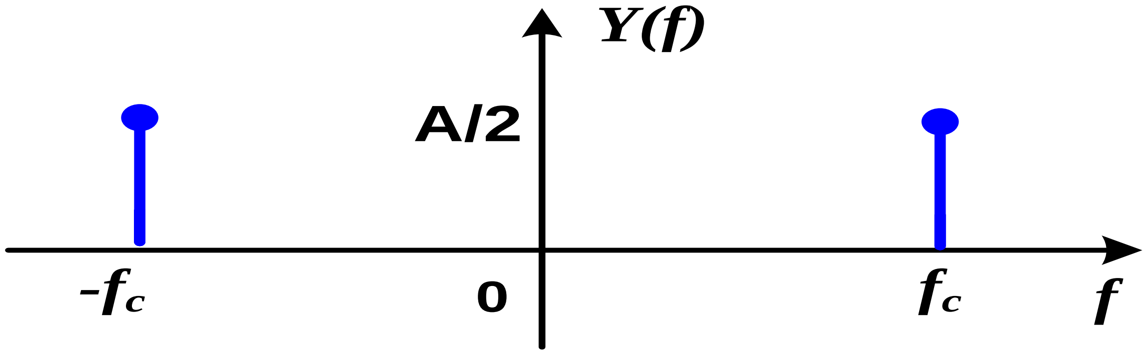


Tentukan $S_A(f)$, $S_B(f)$, $S_B(t)$!

Latihan Soal

3. Diketahui sinyal dalam domain frekuensi sebagai berikut:





a. Untuk fc > fm, Gambarkan $Z(f) = X(f) * Y(f)$!
b. Tentukan persamaan z(t), gambar diagram proses yang terjadi !