

SISTEM KOMUNIKASI 2

***SPREAD SPECTRUM
DAN CDMA***

**Program Studi S1 Teknik Telekomunikasi
Fakultas Teknik Elektro – Telkom University
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- Historically spread spectrum (SS) system has been used by military since over a half century ago for two main purpose:
 - To overcome strong intentional interference (jamming)
 - To hide signal from the eavesdropper.
- Most papers/text book prior to the end of 1980s emphasise this application of SS system. Only at the beginning of 1990s the subject of SS systems for commercial applications began to gain much attention.
- In fact, SS system for commercial applications can achieve efficiency improvements by incorporating a number of unique features made possible by the benign noise-like characteristics of the SS signal waveform.
 - Universal frequency reuse → increase spectrum efficiency
 - Mitigation of multipath fading → Rake receiver
 - Soft hand-off in multiple cells → improve cell boundary performance and prevents dropped calls

- Limitations of conventional FDMA and TDMA:
 - Each channel is allocated a disjoint freq or time slot that is orthogonal (assuming perfect isolation).
 - Channel capacity is limited by BW and time allotment, thermal AWGN, and propagation effect (shadowing and multipath fading)
 - This permanent channel allocation is good for continuous transmission, in fact voice transmission is not continuous.
 - Frequency reuse needs a very careful design because of potential cochannel interference.
 - FDMA and TDMA suffer degradation due to multipath fading
- Benefit of SS multiple access is that it can overcome most of the limitations of conventional systems.

| Purposes | Military | Commercial |
|------------------------|-----------------|-------------------|
| antijamming | yes | Yes |
| Multiple access | yes | Yes |
| Low Prob. Of Intercept | yes | No |
| Message privacy | yes | Yes |
| Selective calling | yes | Yes |
| identification | yes | Yes |
| Navigation | yes | Yes |
| Multipath protection | yes | Yes |
| Low power/flux density | yes | yes |

$$C = B_{\omega} \log_2 \left[1 + \frac{S}{N} \right]$$

■ Dimana:

C= Kapasitas kanal transmisi (bps)

B_ω= Lebar pita frekuensi transmisi (Hz)

S= Daya Sinyal (watt)

N= Daya derau (watt)

- Menyalurkan informasi yang jauh lebih besar (Kapasitas kanal transmisi besar) pada saluran ber-noise dapat ditempuh dengan 2 cara, yaitu:
 1. Dengan cara konvensional, dimana **B_ω** kecil dan **S/N** besar
 2. Dengan cara penyebaran spektrum, dimana **B_ω** besar dan **S/N** kecil
- Pada sistem spektral tersebar sinyal informasi disebar pada pita frekuensi yang jauh lebih besar daripada pita informasinya.
- Penyebaran ini dilakukan oleh suatu fungsi penebar yang bebas terhadap sinyal informasi berupa sinyal acak semu (*pseudorandom*) yang memiliki karakteristik spektral mirip derau (*noise*), disebut pseudorandom noise (*PN Code*).

Jenis-jenis Spread Spectrum

- **Averaging System:** sinyal di-spread pada keseluruhan band width sangat lebar sepanjang waktu.

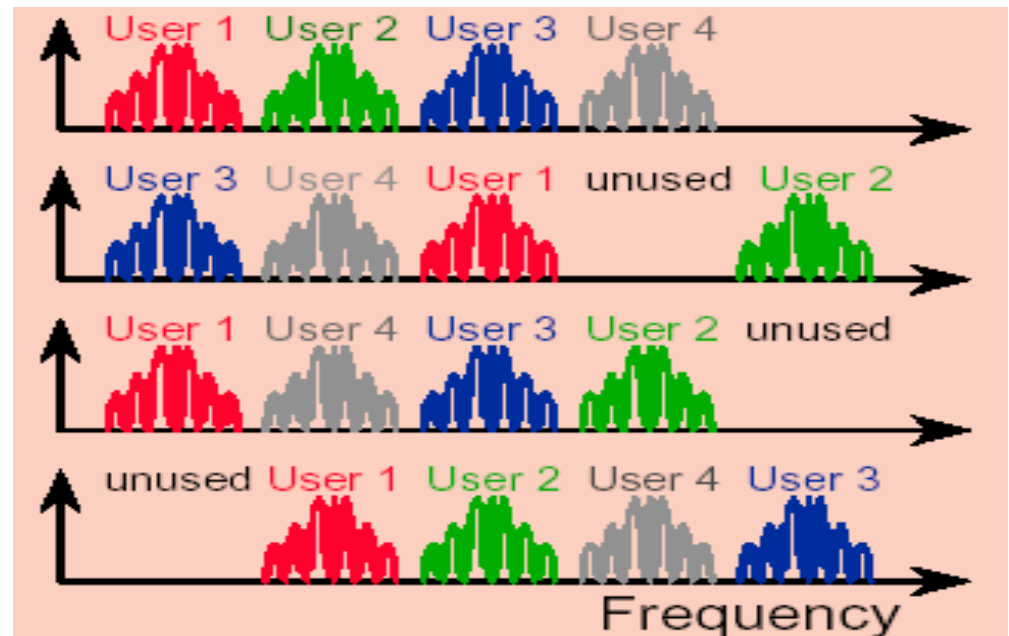
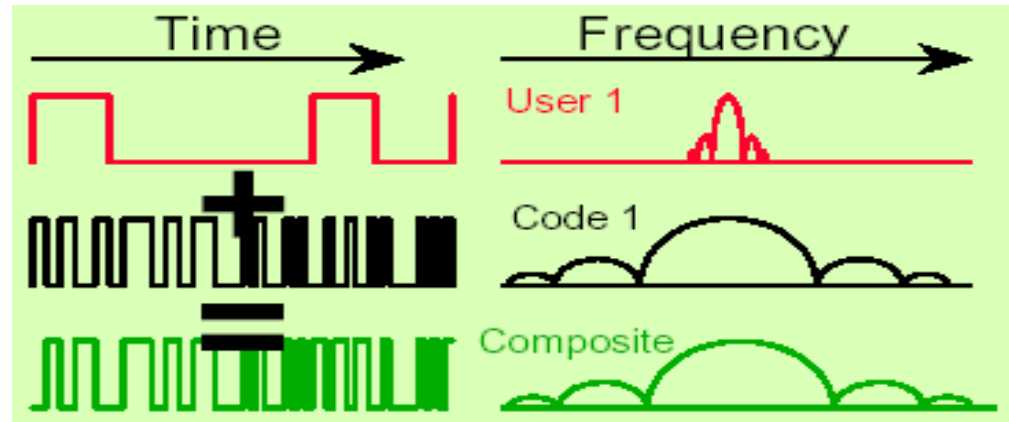
⇒ *Direct Sequence-Spread Spectrum (DS-SS)*

- **Avoidance System:** sinyal modulasi narrow band di-hopped pada band width atau waktu sangat lebar sehingga dapat “menghindari” gangguan.

⇒ *Frequency Hopping-Spread Spectrum (FH-SS)*

⇒ *Time Hopping-Spread Spectrum (TH-SS)*

- **Hybrid :** ⇒ DS/FH, TH/DS, etc.

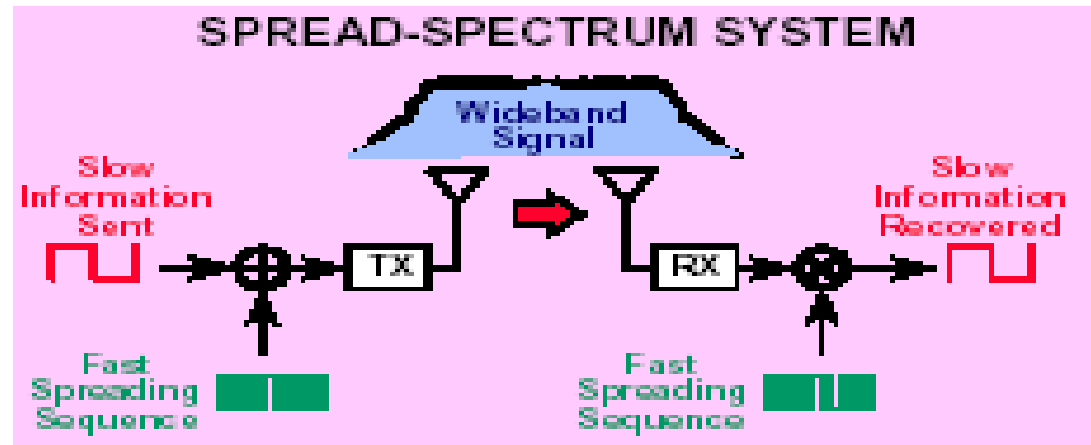


Traditional Communication System Vs Spread Spectrum Systems

- Sistem Komunikasi tradisional mengirimkan urutan data dengan *bandwidth* yang sempit



- Sistem Spread Spectrum mengirimkan urutan data dengan *bandwidth* yang lebar

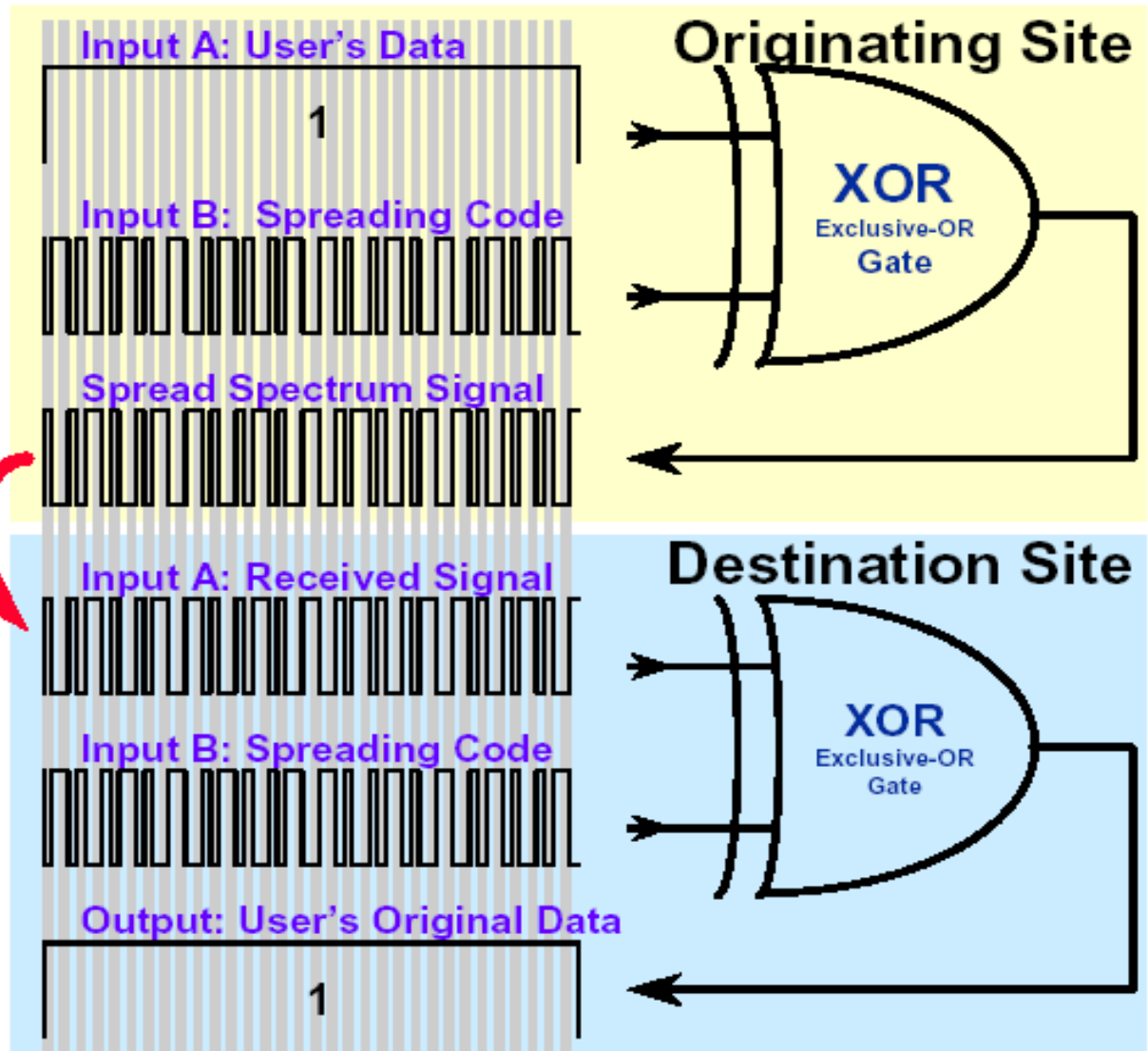


Spread-Despread Principle in DS-SS

- Proses Spread-Despread pada **domain waktu**
- Jika **format data NRZ unipolar**
- Dengan gerbang XOR

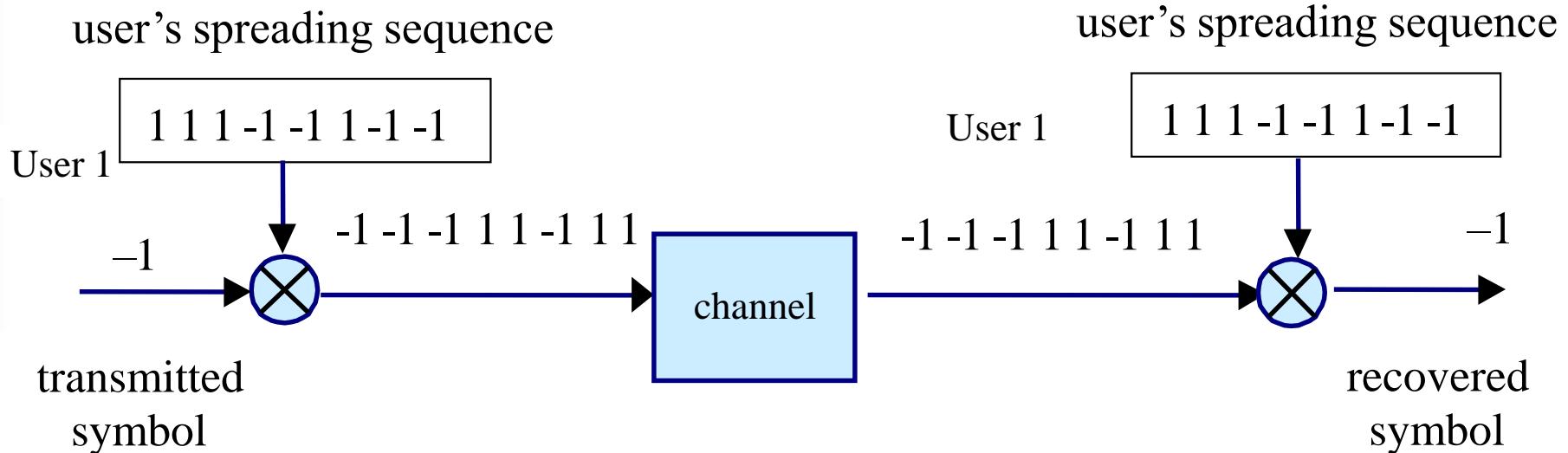
Lewat kanal transmisi

| A | B | $A \oplus B$ |
|---|---|--------------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



Spread-Despread Principle in DS-SS

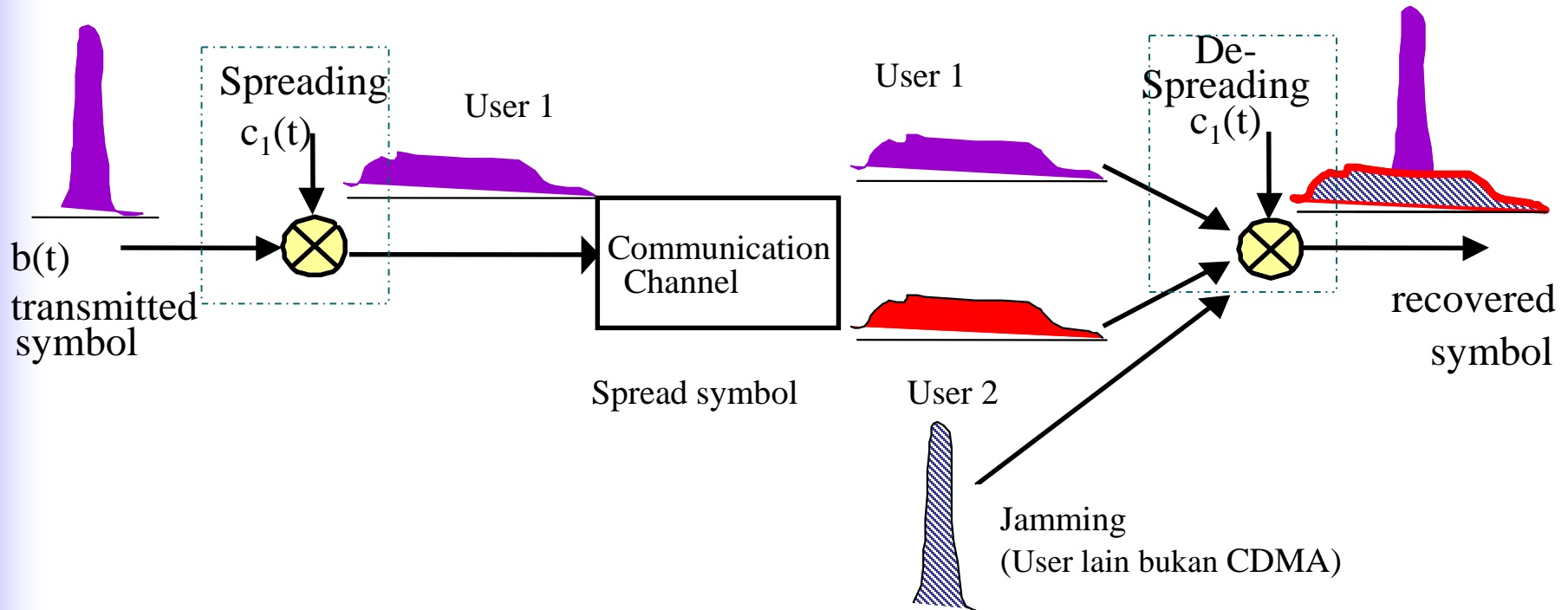
Proses Spread-Despread pada domain waktu, jika format data NRZ bipolar \Rightarrow proses dengan operasi pengali



- The spreading sequence (code) is used at both the transmitter and receiver.
- The code must have good correlation property (low crosscorrelation, orthogonal, regenerative, easy to synchronize).
- There are many types of code (m-sequence, Gold, Walsh-Hadamard, etc)

Spread-Despread Principle in DS-SS

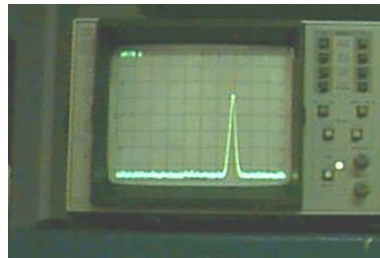
Proses Spread-Despread pada domain frekuensi



- The desired user despread the received signal
- Spread and despread with the matched code results in detection
- Spread and despread with the wrong code results in interference
- Despread only at the receiver results in suppressed interference

Characteristics of Spread Spectrum

- Bandwidth of the transmitted signal W is much greater than the original message bandwidth (or the signaling rate R)
- Transmission bandwidth is independent of the message. Applied code is known both to the transmitter and receiver



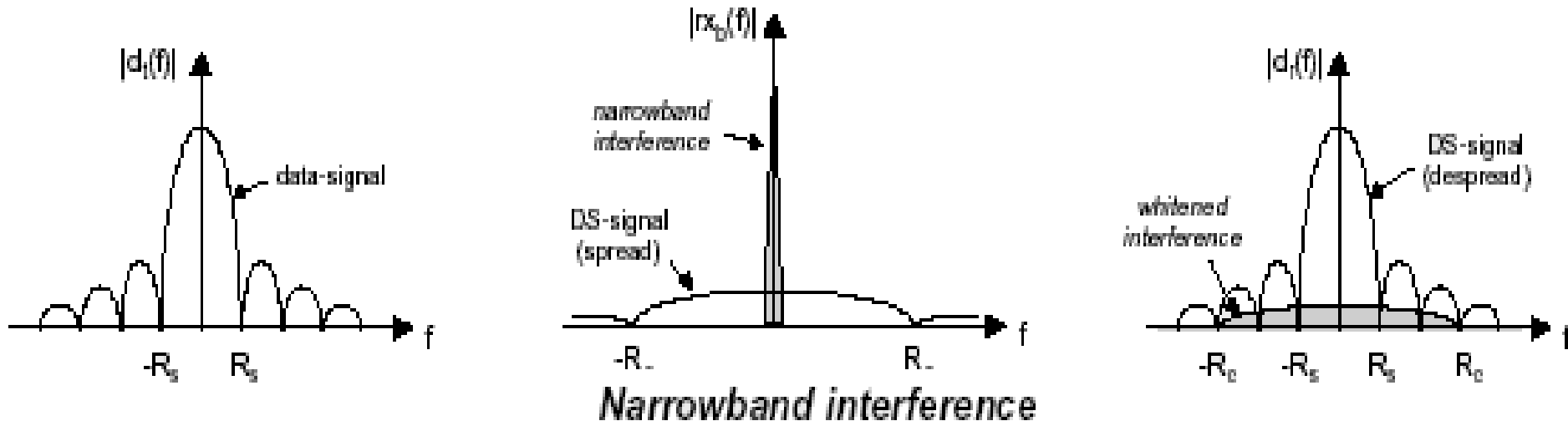
Narrow band signal
(data)



Wideband signal
(transmitted SS signal)

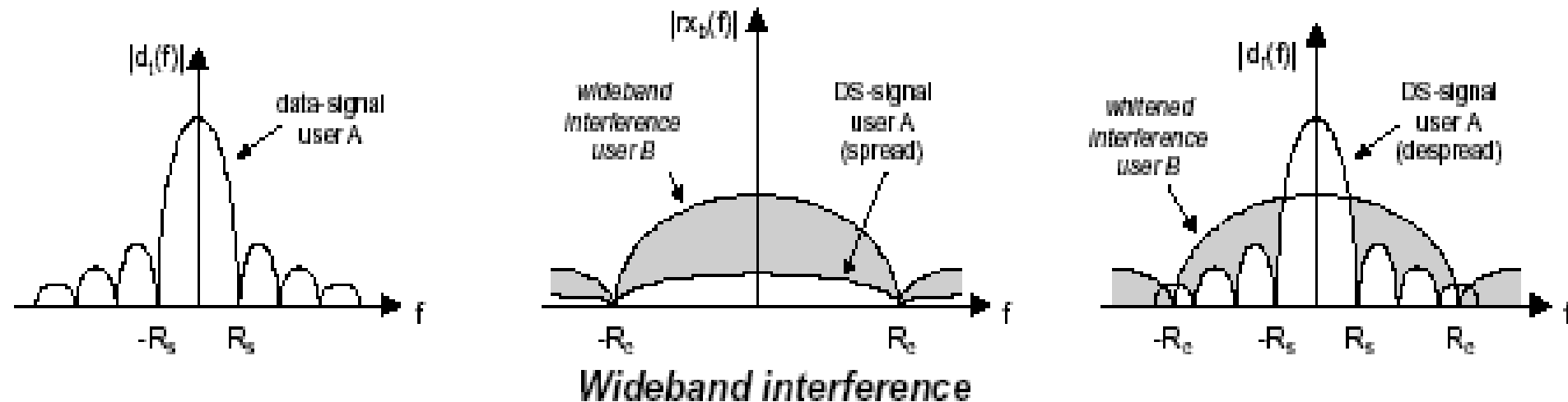
- Interference and noise immunity of SS system is larger, the larger the **processing gain** $L_c = W / R = T_b / T_c$
- Multiple SS systems can co-exist in the same band (=CDMA). Increased user independence (decreased interference) for (1) **higher processing gain** and higher (2) **code orthogonality**
- Spreading sequence can be very long -> enables low transmitted PSD-> **low probability of interception** (especially in military communications)

- Kemampuan untuk menekan jamming (Narrowband interferer)



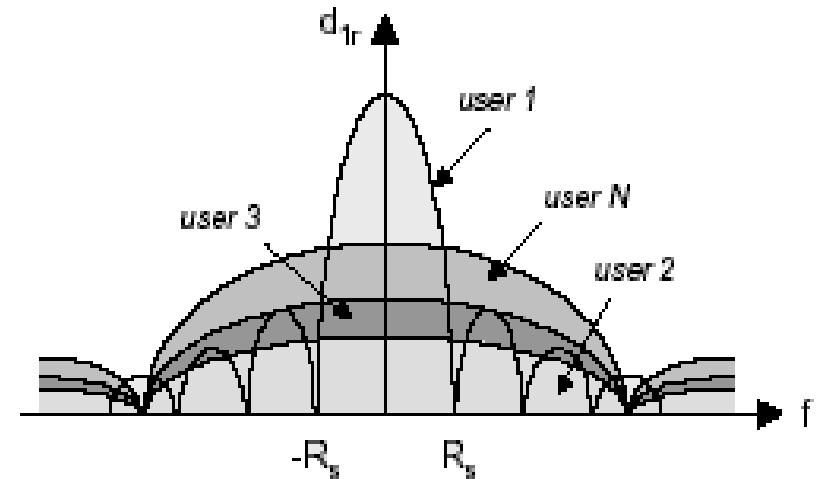
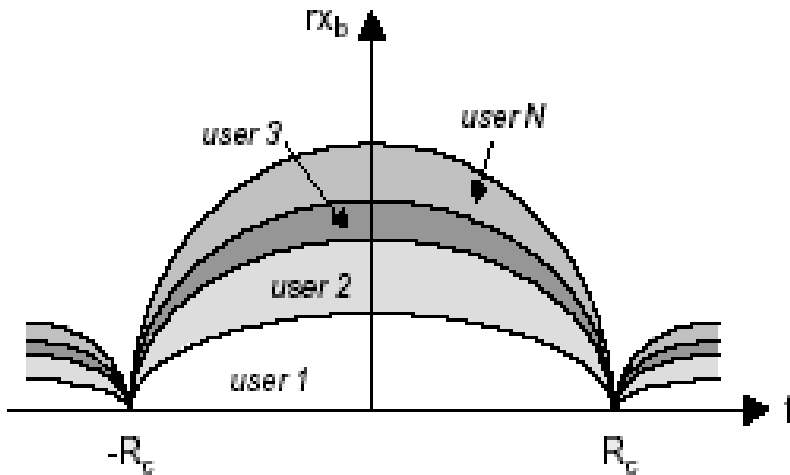
- Pada sistem DS-SS sinyal informasi menduduki bandwidth yang besar dibandingkan bandwidth aslinya. →lihat gbr kiri & tengah
- Selama transmisi sangat mungkin bercampur dengan jamming (Narrowband interferer) →lihat gbr tengah
- Setelah melewati de-Spreader, sinyal informasi akan terkumpul kembali, sedangkan jamming akan tersebar →lihat gbr kanan

- Tahan wideband interferer



- Pada sistem DS-SS sinyal informasi menduduki bandwidth yang besar dibandingkan bandwidth aslinya. →lihat gbr kiri & tengah
- Selama transmisi sangat mungkin bercampur dengan *wideband interferer*. →lihat gbr tengah
- Setelah melewati de-Spreader, sinyal informasi akan terkumpul kembali, sedangkan *wideband interferer* akan tersebar/masih lebar →lihat gbr kanan

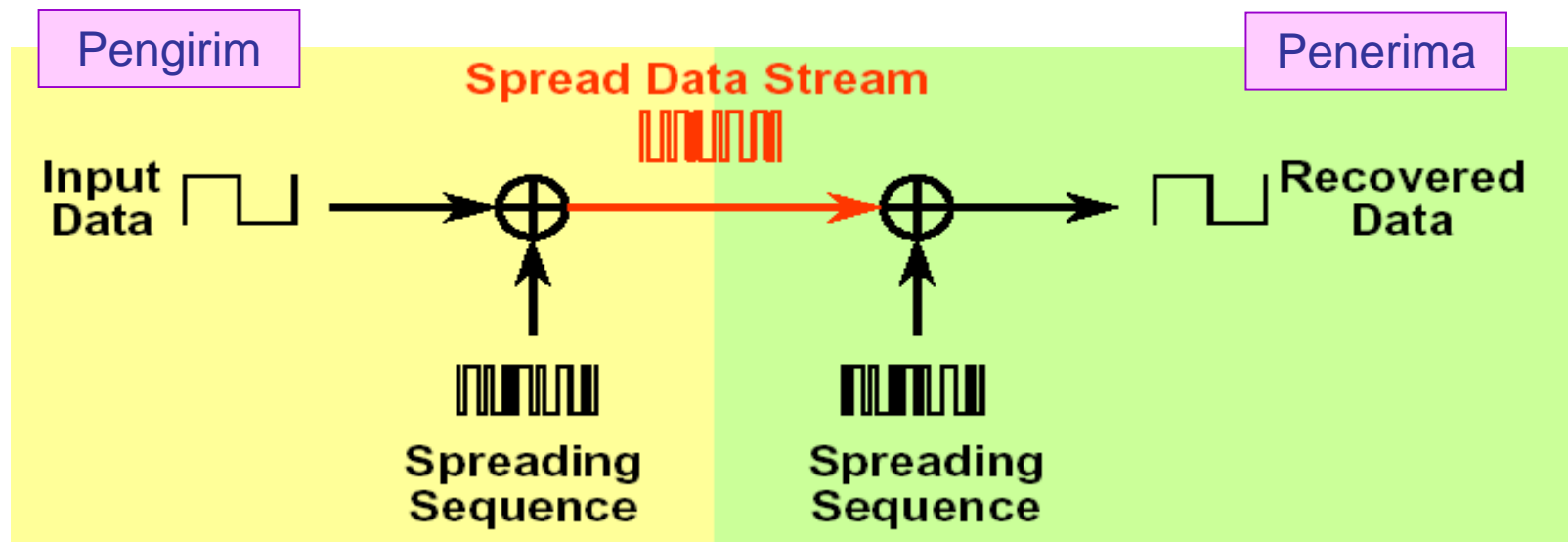
•Kemampuan untuk Multiple Acces (CDMA)



- Pada sistem DS-SS sinyal informasi menduduki bandwidth yang sama untuk banyak user (sejumlah N user). →lihat gbr kiri
- Tiap User dibedakan oleh kode yang berbeda. (\Rightarrow CDMA: *Code Division Multiple Acces*). →lihat gbr kiri
- Jika hanya ingin mendeteksi user ke-1 saja, maka setelah melewati de-Spreader, sinyal informasi user ke-1 akan terkumpul kembali, sedangkan user ke-2 sampai ke-N akan tersebar dan akan menjadi interferer \Rightarrow yang sering disebut *MAI (Multiple Acces Interference)* →lihat gbr kanan

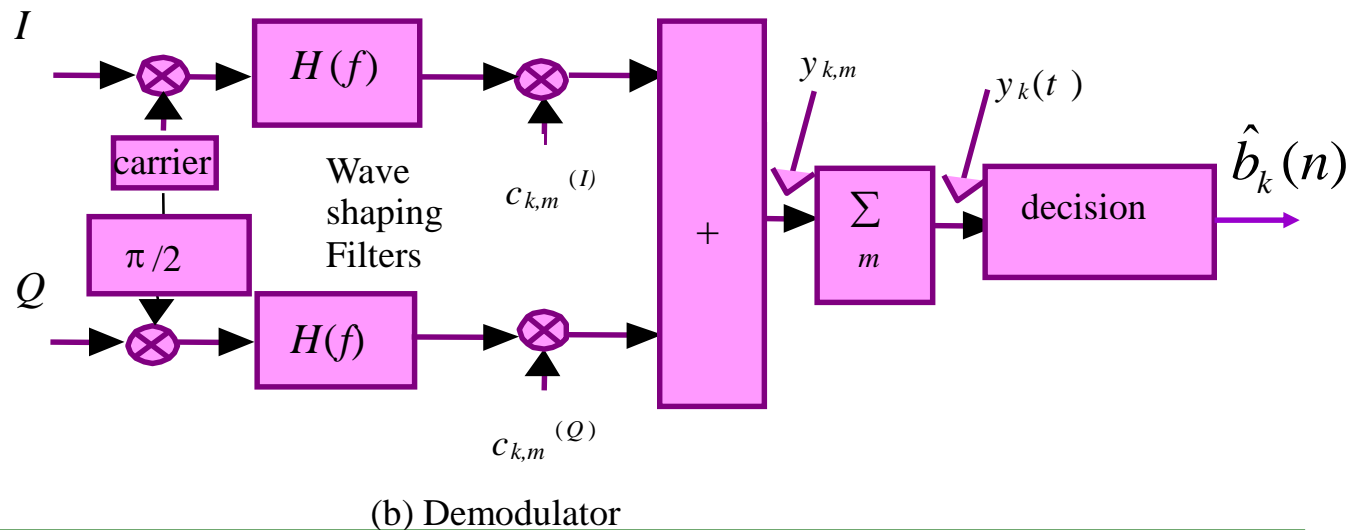
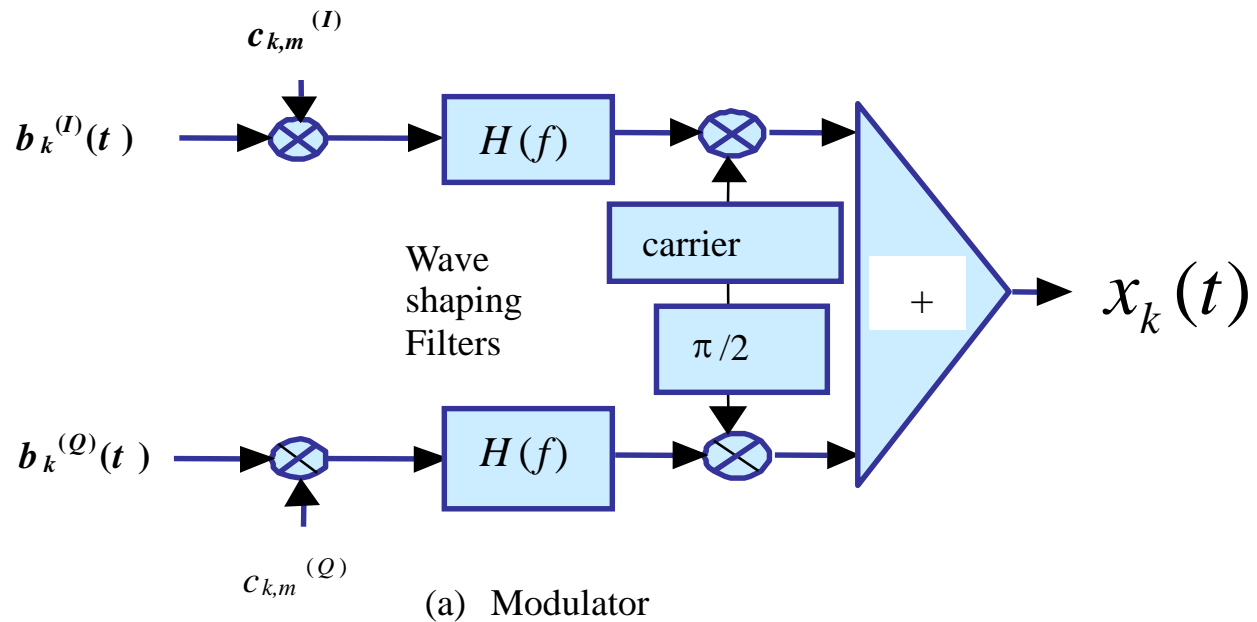
- Jaminan keamanan komunikasi yang tinggi

- Pada sistem DS-SS sinyal informasi menduduki bandwidth yang sama untuk banyak user (sejumlah N user, tiap User dibedakan oleh kode yang berbeda.
- Jika ingin **mendeteksi user ke-1** saja, maka harus digunakan **kode penebar yang sama persis seperti di pengirim** untuk proses de-Spreader/deteksi
- Kode yang lain tidak akan mampu mendeteksi / melakukan de-Spreader



- Characteristics of DS-SS
 - Spreading sequence (Code) is generated to distinguish between different users.
 - Spreading sequence is used to spread (at the transmitter) and to despread (at the receiver) the user's data symbol.
 - Depending on the spreading code used, and the channel conditions (multipath) → multiple access interference (MAI) is the limiting factor that determine the capacity
 - The user can be detected when ratio of signal-to-interference (SIR) is sufficient.

Example in QPSK



- The transmitted signal can be expressed as

$$x_k(t) = \sqrt{2P_k} b_k c_k s_k(t)$$

- P_k = transmit power, b_k = data symbol, c_k =spreading sequence
 s_k = chip waveform, **all of which are for the k^{th} user.**

- The wireless channel can be modeled as

$$h_k(t) = \sum_{l=0}^{L-1} \beta_{k,l} \delta(t - \tau_{k,l}) e^{j\theta_{k,l}}$$

L = the number of multipath, β = channel gain, δ =impulse, θ =channel phase, τ = time delay of multipath channel

- Due to propagation delay, the signal has a delay of $\tau_{k,l}$.
- Therefore the received signal is (when $h(t)$ is the channel)

$$\begin{aligned} r(t) &= \sum_{k=1}^K x_k(t) * h_k(t) \\ &= \sum_{k=1}^K \sum_{l=0}^{L-1} \sqrt{2P_k} b_k c_k \beta_{k,l} s_k(t - \tau_{k,l}) e^{j\theta_{k,l}}. \end{aligned}$$

- At the receiver the copy of spreading sequence waveform $s_k(t)$ is generated and synchronized, that is : $s(t - \tau_{k,l})$, where $\tau_{k,l}$ is time delay of the l^{th} path for the k^{th} user

$$c_k^*(t) = \sum_{m=0}^{M-1} c_{k,m}^* s(t - mT_c), 0 \leq t \leq MT_c$$

- Decision variable after despreading

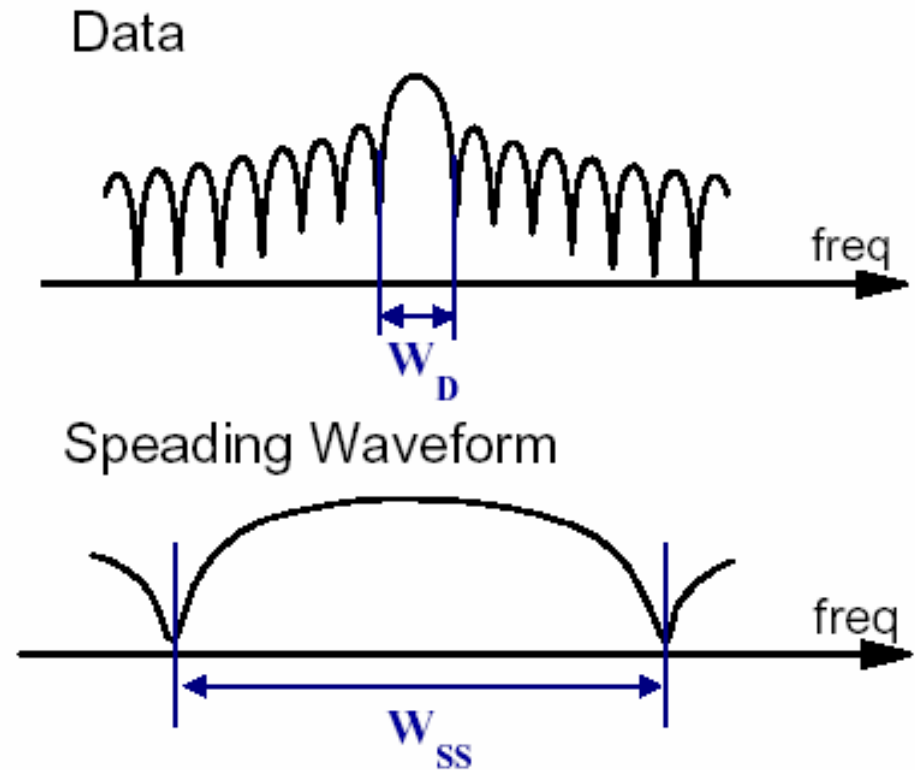
$$y_k(t) = M \sqrt{2P_k} b_k \sum_{l=0}^{L-1} \beta_{k,l} + \sum_{j \neq k} \sum_{m=0}^{M-1} c_{k,m}^* r_{j,m} + \underline{n(t)}$$

- First term is the desired signal (despreading).
- Second term is **multiple access interference** from other user.
- Third term is AWGN
- The desired signal obtain processing gain M
- SIR can be expressed as

$$\gamma_{k,l}(t) = \frac{MP_k(t)\beta_{k,l}^2(t)}{\sum_{j \neq k} P_j(t)\beta_{j,l}^2(t) + \sigma_n^2}$$

- Didefinisikan sebagai perbandingan Bandwidth sinyal Spread Spectrum terhadap Bandwith data

$$M = PG = \frac{W_{SS}}{W_D} = \frac{R_C}{R_D} = \frac{T_D}{T_C}$$



Untuk membentuk sistem CDMA, digunakan kode penebar yang berbeda untuk setiap kanal.

Sifat kode-kode penebar yang digunakan :

- Memiliki panjang kode (periode pengulangan) yang sama
- Bersifat ortogonal/semiorthogonal satu sama lainnya.

Sifat pertama (periode kode):

Misalkan kode PN = 111110010110001

ini adalah sebuah kode PN dengan panjang kode = $L = 15$. Semua kode yang lain juga harus memiliki panjang kode = 15.

Sifat kedua (ortogonalitas) :

Misalkan kita miliki dua buah kode penebar dengan panjang sama (untuk memenuhi syarat pertama).

Contoh : dua kode dengan panjang $L = 8$

Kode pertama = $\{ b_1 \} = 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0$

Kode kedua = $\{ b_2 \} = 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1$

Urutan $\{ b_1 \}$ dapat diasosiasikan dengan urutan $\{ a_1 \}$, dimana :

$$\{ a_1 \} = 1 \ 1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1$$

Urutan $\{ b_2 \}$ dapat diasosiasikan dengan urutan $\{ a_2 \}$, dimana :

$$\{ a_2 \} = -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ 1$$

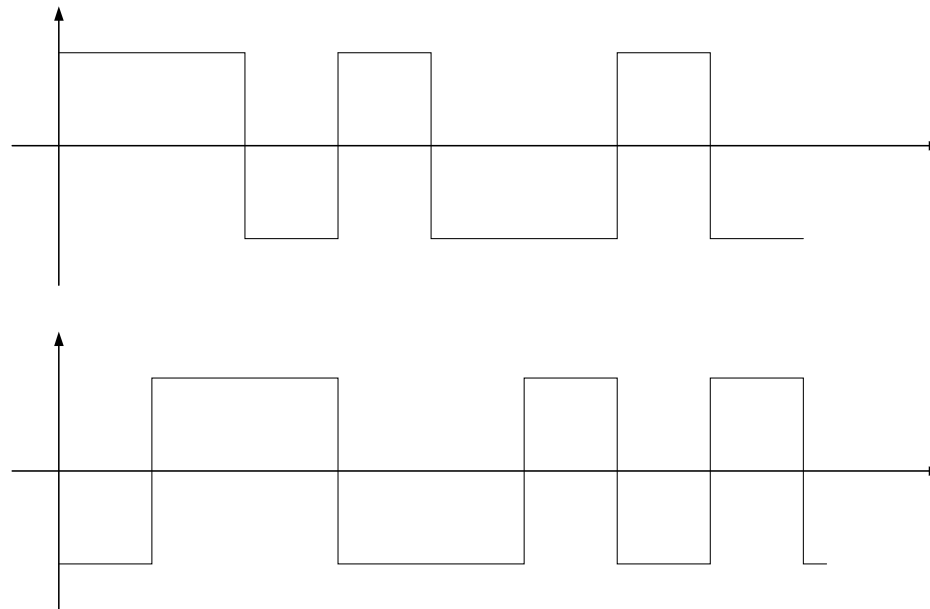
Apabila kedua kode penebar diatas bersifat ortogonal satu sama lain, maka :

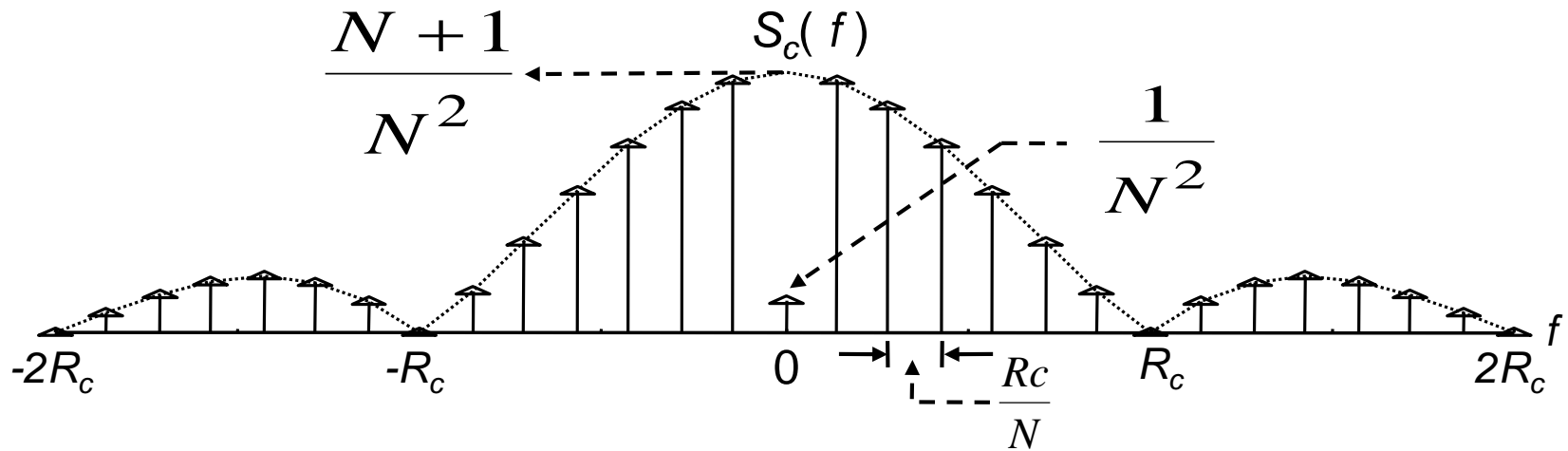
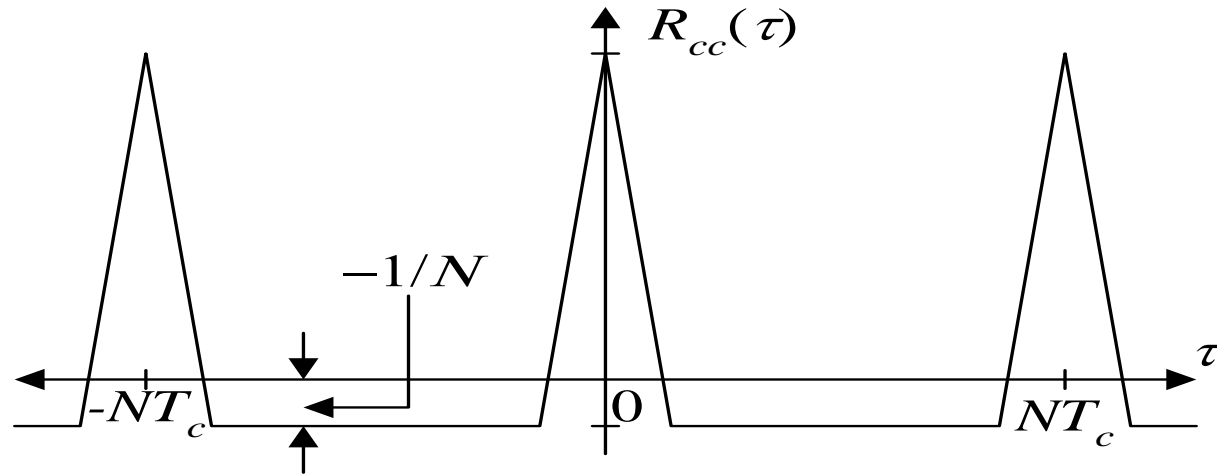
$$\sum_{j=1}^L \mathbf{a}_{1j} \mathbf{a}_{2j} = \mathbf{0} \quad \text{atau} \quad \int_0^{LT_c} a_1(t) a_2(t) dt = \mathbf{0}$$

Kode pseudo noise (PN) paling banyak digunakan dengan sifat *semiorthogonal* akibat *crosscorrelasi* antara dua kode yang berbeda tidak sama dengan nol \rightarrow mudah dibangkitkan

Sifat ortogonal dapat diselidiki dari *sinyal penebar* yang mewakili tiap kode.

Contoh :





Kode yang dihasilkan oleh sebuah susunan shift register dengan feedback bergantung pada :

- Jumlah register (elemen flip-flop) yang digunakan
- Konfigurasi dari sambungan feedback (jumlah adder modulo-dua)
- Kondisi awal (state awal) dari register-register.

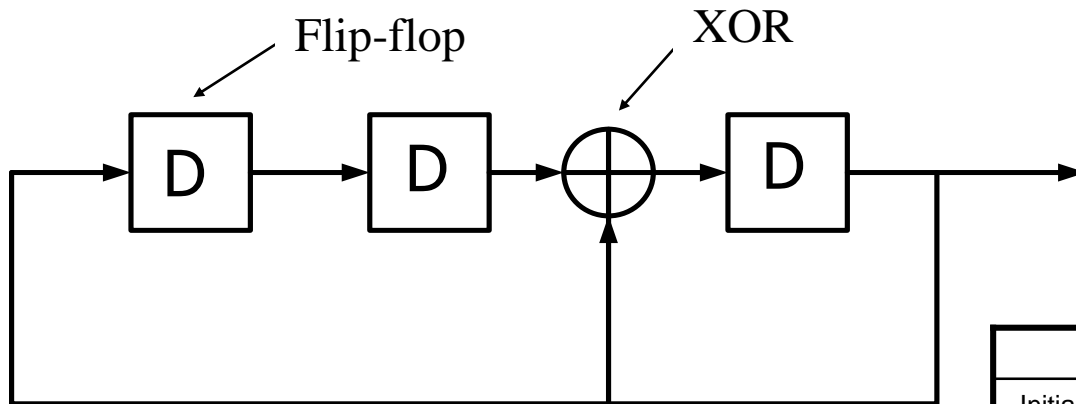
Dengan memasukkan kondisi awal pada tiap register, kemudian memasukkan pulsa clock (synchronous clock) pada semua register, keluaran sistem shift register ini dapat diketahui dengan mengurutkan sinyal keluaran tiap blok.

Untuk keperluan analisis , sistem shift register dengan feedback dapat diasosiasikan dengan sebuah polinom “generator” $g(D)$.

Pada contoh diatas : $g(D) = 1 + D + D^3$

Rangkaian yang umum digunakan untuk membangkitkan kode PN adalah susunan shift register dengan feedback.

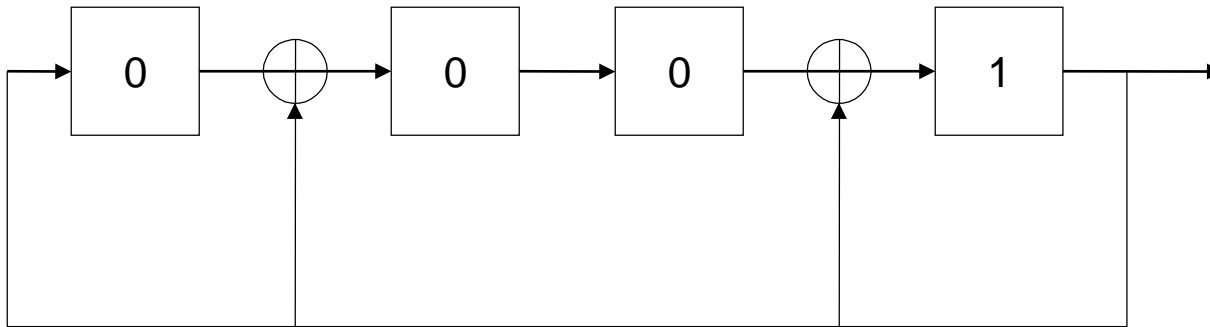
Contoh : $g(D) = 1 + D + D^3$



Urutan keluaran dengan initial state 111:

| | | Stage 1 | Stage 2 | Stage 3 |
|------------------|---|---------|---------|---------|
| Initial contents | | 1 | 1 | 1 |
| Clock pulse | 1 | | | |
| | 2 | | | |
| | 3 | | | |
| | 4 | | | |
| | 5 | | | |
| | 6 | | | |
| New period | 7 | | | |

Contoh lain:



Polinom generator dari sistem diatas : $g(D) = 1 + D + D^3 + D^4$

State awal dari register dapat dinyatakan dengan polinom :

$$a(D) = 1 + 0.D + 0.D^2 + 0.D^3 = 1$$

Untuk menentukan keluaran dari sistem, bisa juga dengan menyelesaikan hasil bagi dari : $1 / (1 + D + D^3 + D^4)$

$$\begin{array}{r}
 1 + D + D^2 + \\
 \phantom{D^{10}} \phantom{D^{11}} \phantom{D^{12}} \\
 \hline
 1 + D + D^3 + D^4 \quad \left| \begin{array}{l} 1 \\ 1 + D \\ D \\ D + D^2 \\ \hline D^2 + D^3 \\ D^2 + D^3 \\ \hline D^6 \\ D^6 + D^7 \phantom{+ D^{10}} \\ \hline D^7 \phantom{+ D^{10}} \\ D^7 + D^8 \phantom{+ D^{10}} \phantom{+ D^{11}} \\ \hline D^8 + D^9 \phantom{+ D^{10}} \phantom{+ D^{11}} \\ D^8 + D^9 \phantom{+ D^{10}} \phantom{+ D^{11}} \phantom{+ D^{12}} \\ \hline D^{12} \end{array} \right.
 \end{array}$$

- Berdasarkan konfigurasi tertentu, satu sistem dengan r buah register akan dapat menghasilkan satu urutan sepanjang $L = 2^r - 1$. Urutan keluaran ini disebut sebagai *Maximum Length Sequence* (MLS).
- Konfigurasi lainnya akan menghasilkan beberapa urutan *bukan MLS* dengan panjang lebih kecil dari $2^r - 1$. Urutan yang dikeluarkan akan bergantung pada state awal yang diberikan pada register-register.
- Dengan memilih konfigurasi sistem yang sesuai, akan diperoleh urutan-urutan dengan panjang yang sama \rightarrow shift register akan menjalani salah satu dari beberapa siklus, bergantung dari state awal yang diberikan.

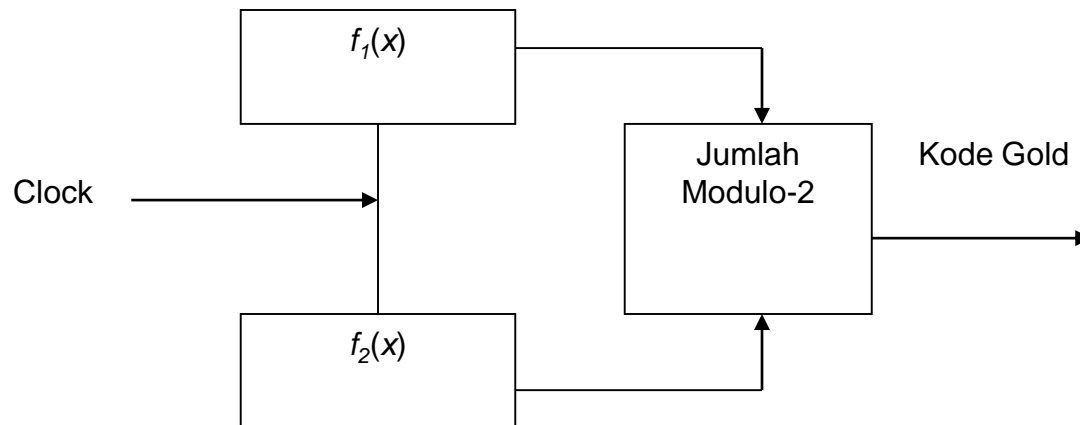
Contoh : (Kode GOLD)

Polinom generator : $1 + D^3 + D^5 + D^6 + D^8 + D^{11} + D^{12}$

Shift Register akan mengeluarkan 65 siklus sama panjang dengan masing-masing siklus memiliki panjang kode $L = 63$.

Gambarkan blok diagram dari Polinom di atas dalam bentuk Linear feedback Shift Register sebagai latihan.

- Dua buah LFSR: $f_1(x) = x^3 + x + 1$, dan $f_2(x) = x^3 + x^2 + 1$ utk menghasilkan Kode Gold
- Gambarkan LFSR utk masing-masing f_1 dan f_2 serta utk kondisi register awal pada $f_1 = 0\ 0\ 0$ dan pada $f_2 = 1\ 1\ 1$ tentukan semua kemungkinan Kode Gold yg dihasilkan



Kode orthogonal bisa dibangkitkan, misalnya kode Walsh-Hadamard

Apabila pasangan kode $\{ a_1 \}$ dan $\{ a_2 \}$ ortogonal,

$$\text{maka : } \int_0^{LT_c} a_1(t) a_2(t) dt = 0$$

Pembangkitan kode Walsh - Hadamard

$$H_1 = 0$$

$$H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & \overline{H_1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & \overline{H_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

.....

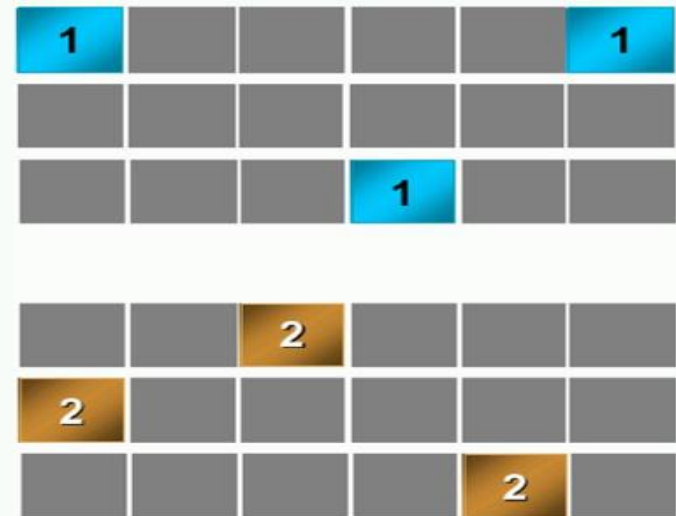
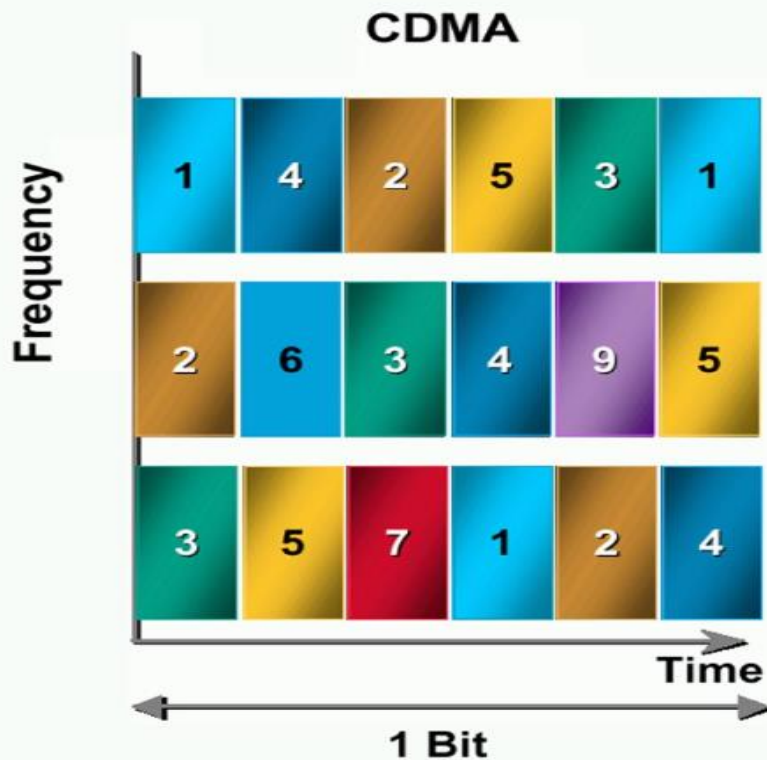
$$H_{128} = 128 \times 128 \text{ Matrix}$$

Lat: Bangkitkan kode Walsh-Hadamard jika $H_1 = 1$

Frequency Hopping SS

- q Classified in Avoidance System
- q 2 techniques : Slow Hop & Fast Hop
- q Very Complex Frequency Synthesizer
- q Non-coherent Modulation : M-ary FSK

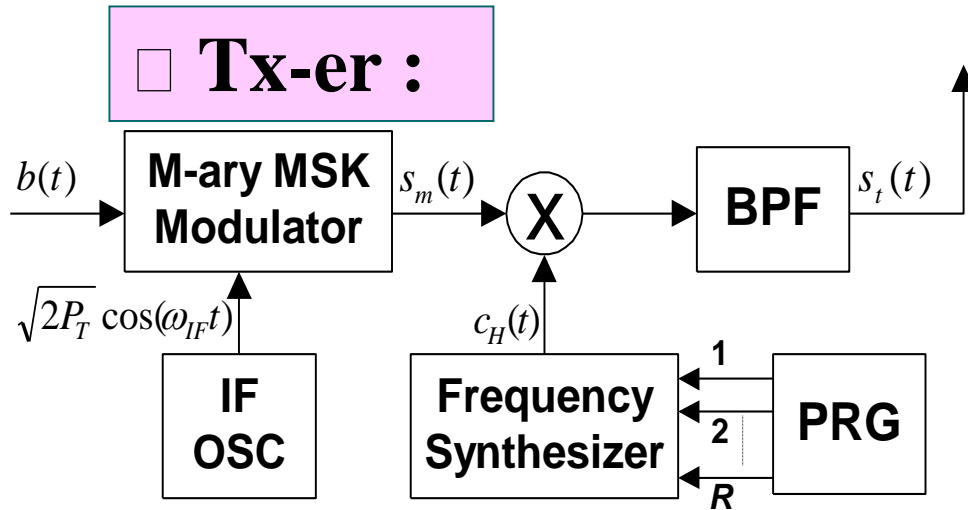
⇒ Fast Frequency Hopping ($R_c > R_b$)



A long code is used to generate a mask for each radio.

Typically a code is transmitted on a separate channel

Tx-er :



$$s_m(t) = \sqrt{2P_T} \cos[\omega_i t]$$

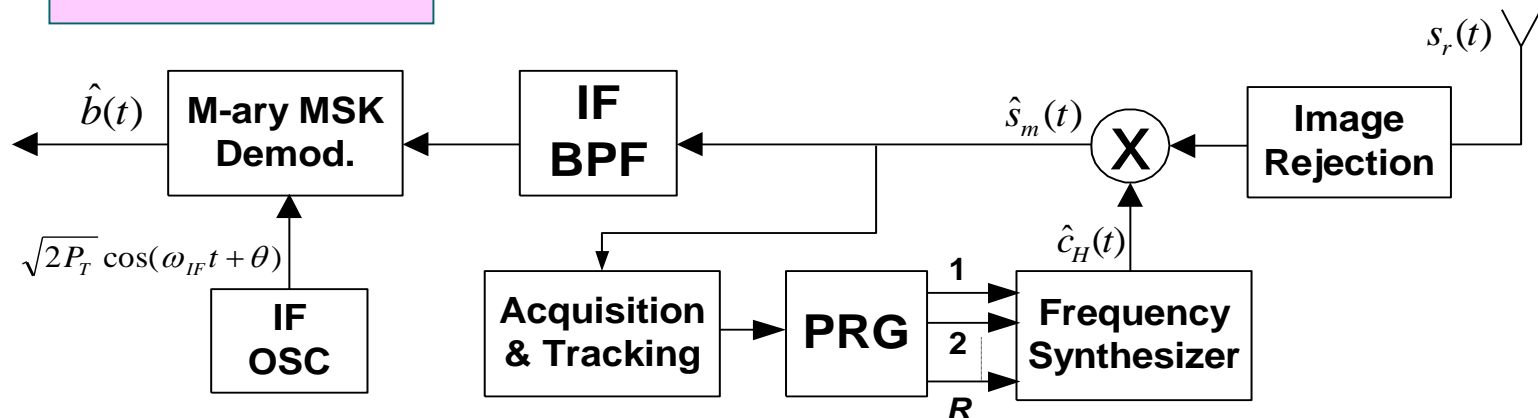
$$; i \in \{1, 2, \dots, M\}$$

$$c_H(t) = 2 \sum_{n=-\infty}^{\infty} p(t - nT_c) \cos(\omega_n t + \varphi_n)$$

$$; \omega_n \in \{\omega_1, \omega_2, \dots, \omega_L\}$$

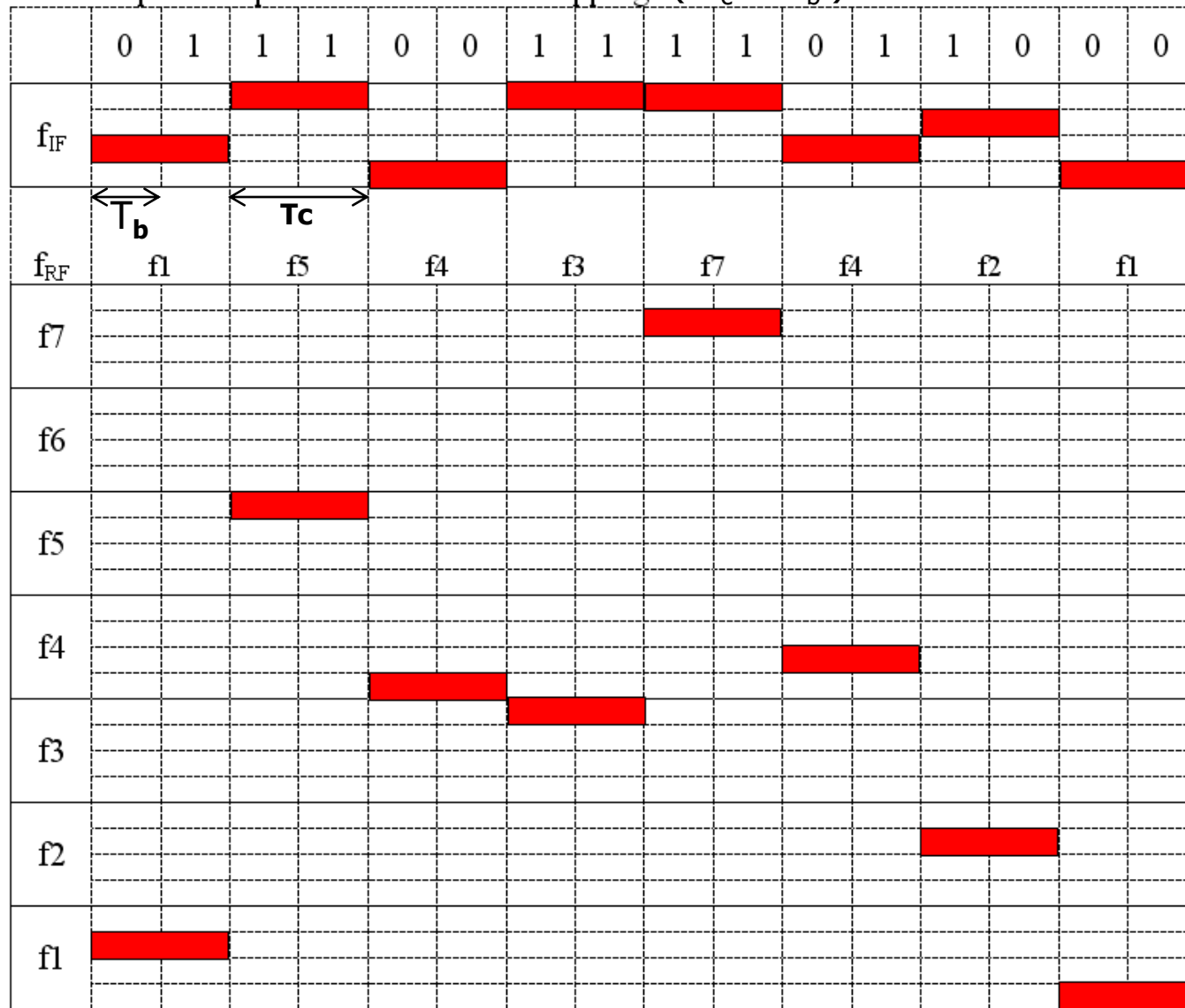
$$; \varphi_n \text{ uncertain}$$

Rx-er :



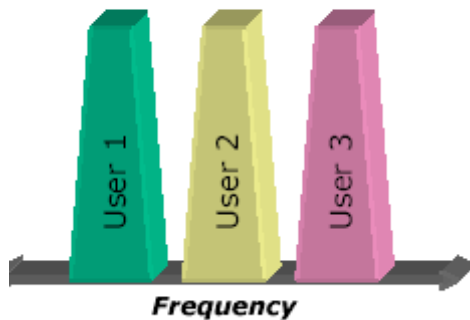
Ilustrasi CDMA Frequency Hopping, modulasi 4FSK MLS[13] :

Simulasi pola lompatan slow frekuensi hopping: ($R_c < R_b$)



Multiple Access Approaches

Frequency
Division
Multiple
Access



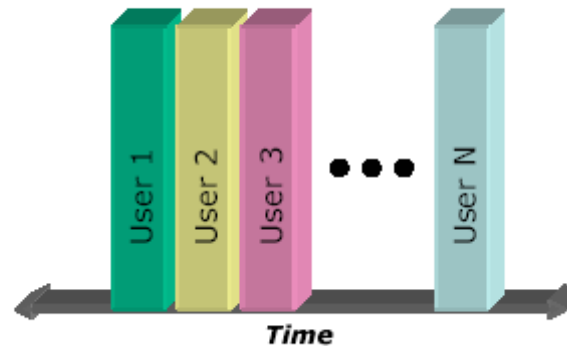
Each User has a unique **frequency**

(1 voice channel per user)

All users transmit at the same time

AMPS, NMT, TACS

Time
Division
Multiple
Access



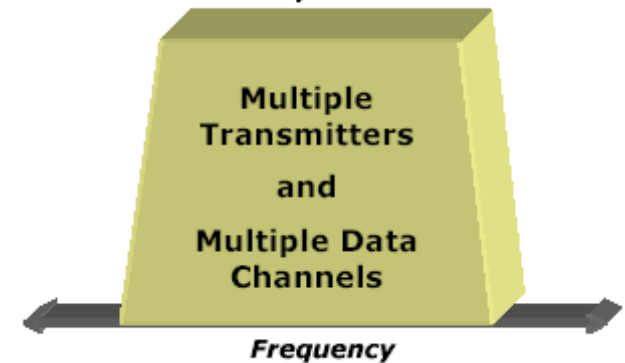
Each User has a unique **time slot**

Each Data Channel has a unique **position within the time slot**

Several users share the same frequency

IS-136, GSM, PDC

Spread
Spectrum
Multiple
Access / **C**ode
Division
Multiple
Access

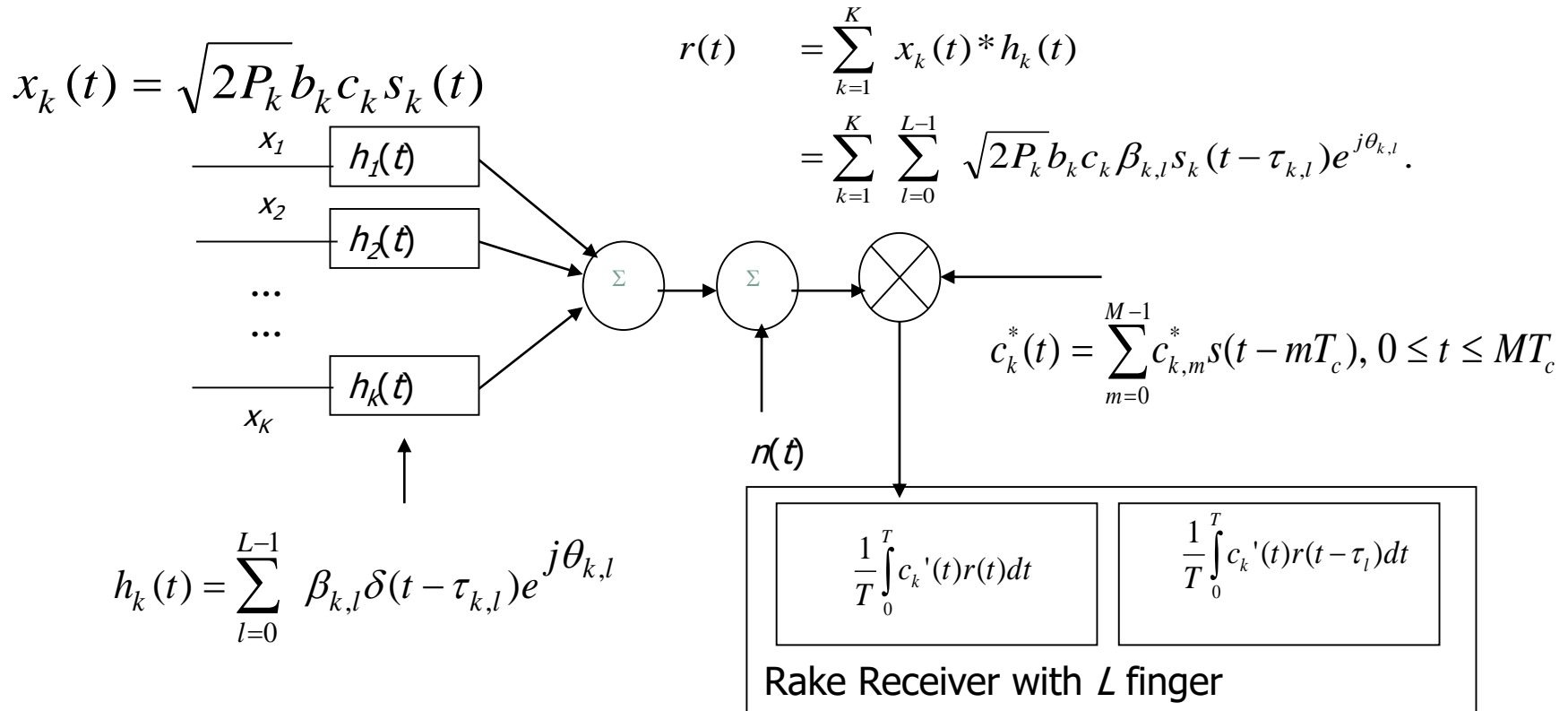


Each Transmitter has a unique **scrambling code (PN code)**

Each Data Channel has a unique **orthogonal code (Walsh code)**

Many users share the same frequency *and* time

IS-95, cdma2000, WCDMA



$$y_k(n) = M \sqrt{2P_k} b_k \sum_{l=0}^{L-1} \beta_{k,l} + \sum_{j \neq k} \sum_{m=0}^{M-1} c_{k,m}^* r_{j,m} + n(t)$$

- P_k transmitted power from the k th user
- b_k transmitted symbol of the k th use
- c_k spreading sequence of the k th user
- $s_k(t)$ spreading waveform
- h_k channel response of the k th user
- $\beta(t)$ channel amplitude response
- θ channel phase response

- L number of resolvable path of channel
- δ unit delta function
- M number of spreading chips per symbol
- $n(t)$ AWGN
- T symbol period
- K number of users

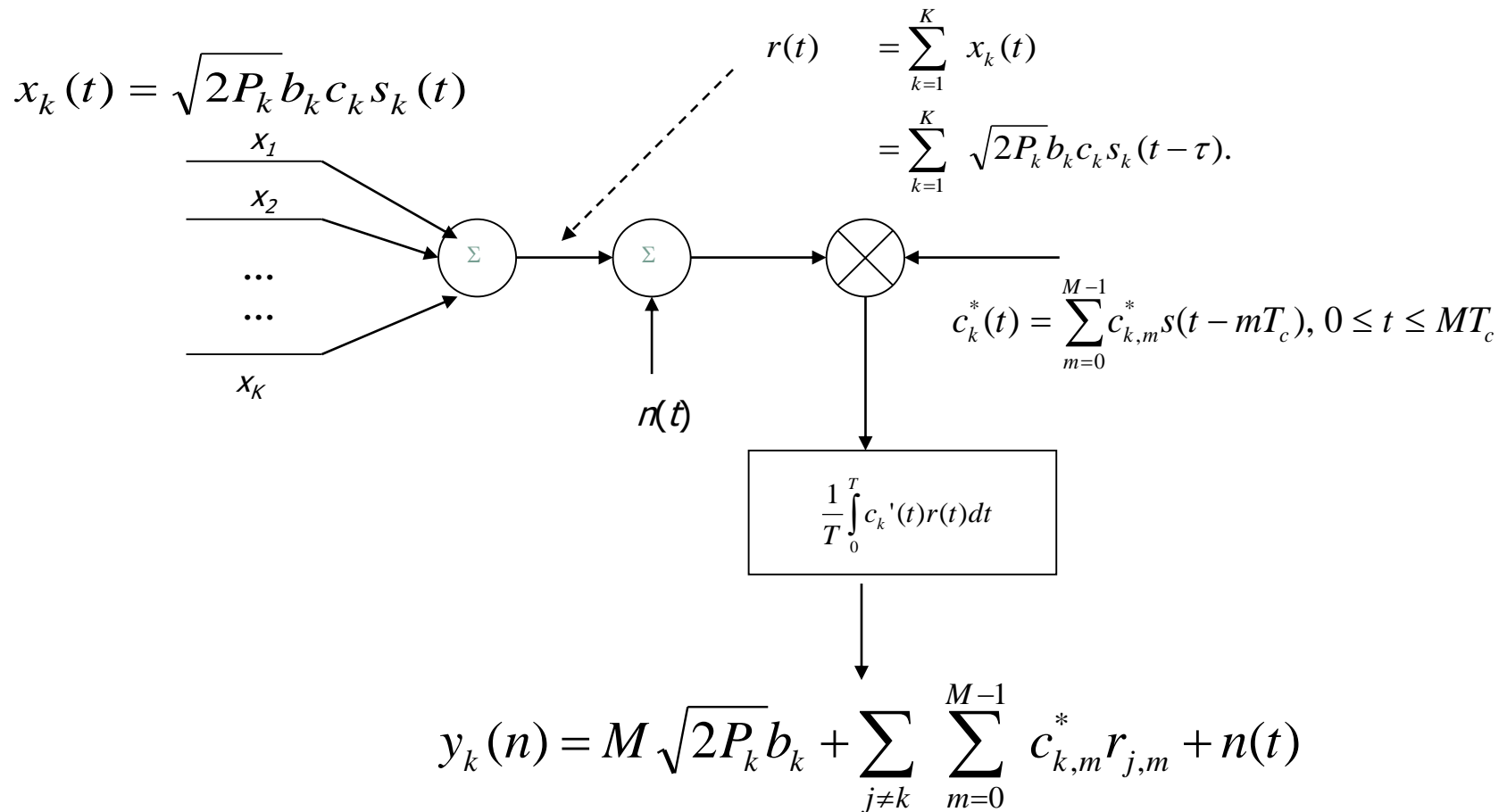
SIR at the l th Rake finger

$$\gamma_{k,l}(n) = \frac{MP_k(n)\beta_{k,l}^2(n)}{\sum_{j \neq k} P_j(n)\beta_{j,l}^2(n) + \sigma_n^2}$$

SIR at the output (MRC)

$$\gamma_k(n) = \sum_{l=0}^{L-1} \gamma_{k,l}$$

Multiple Access Model



$$SINR_k(n) = \frac{MP_k(n)}{\sum_{j \neq k} P_j(n) + \sigma_n^2}$$

Here M = processing gain

σ_n^2 = AWGN variance

P_k = received signal power of the k^{th} user

K = number of user

$$\frac{E_b}{I_0} = \frac{P/R}{[\sigma_n^2 + (K-1)P]/W}$$

The number of users for this case is -

$$K = 1 + \left[\frac{W}{R} \left\{ \frac{1}{(E_b/I_0)} - \frac{1}{SNR} \right\} \right]$$

$$\text{If } K = 1 \rightarrow (E_b/I_0) = SNR = [\sigma_n^2/P]^{-1}$$

Example 1

1. Spread spectrum is used to transmit a single channel military data at a bit rate of $R = 64$ kbps. If a wide band spreading sequence is used in which the chips rate is 3.6864 Mcps and the BPSK receiver operates at $E_b/I_0 = 7$ dB to achieve the minimum required bit error rate of 10^{-3} . Receiver thermal noise contributes to the SNR after despreading of 10 dB. Calculate the anti jamming margin of this system.
- Spektral tersebar dipergunakan untuk transmisi satu kanal data militer pada bit rate 64 kbps. Jika menggunakan deretan kode dengan laju chip pada 3,6864 Mcps dan penerima BPSK bekerja pada $E_b/I_0 = 7$ dB untuk mencapai keperluan BER minimum 10^{-3} . Penerima berkontribusi derau thermal setelah proses despreading sehingga SNR = 10 dB. Hitung margin untuk anti jamming.

Solution for example 1

1.
$$\frac{E_b}{I_0} = \frac{P/R}{[\sigma_n^2 + (K-1)P]/W}$$

Only one user \rightarrow plus one jamming user with P_j

$$\frac{E_b}{I_0} = \frac{P/R}{[\sigma_n^2 + P_j]/W} = \frac{W}{R} \frac{P}{\rho_n^2 + P_j} = (57.6) \frac{1}{\frac{P_j}{P} + \frac{1}{SNR}}$$

$$17.6[dB] - 10.1 \log\left\{\frac{P_j}{P} + 0.1\right\} = \frac{E_b}{I_0} [dB]$$

$$10.1 \log\left\{\frac{P_j}{P} + 0.1\right\} = 17.6[dB] - \frac{E_b}{I_0} [dB] = 11.6[dB]$$

$$10.1 \log\left\{\frac{P_j}{P} + 0.1\right\} = 11.5[dB]$$

$$\frac{P_j}{P} + 0.1 = 10^{1.15} = 14.1254$$

$$\frac{P_j}{P} = 14.0254 = 11.4691[dB]$$

Example 2

1. Let the information bit rate is $R = 8.2$ kbps and the spread bandwidth is $W = 1.25$ MHz. If the receiver contribute thermal noise so that the output $SNR = 10$ dB and the demodulator requires $E_b/I_0 = 7$ dB to achieve a bit error rate $BER = 10^{-3}$. How many user can be served by this system.
- Bit rate informasi adalah $R = 8.2$ kbps dan lebar pita tersebar adalah $W = 1.25$ MHz. Jika penerima menimbulkan derau thermal sehingga $SNR = 10$ dB dan demodulator membutuhkan $E_b/I_0 = 7$ dB untuk mencapai $BER = 10^{-3}$. Berapa jumlah user yang dapat dilayani oleh sistem ini.

$$K = 1 + \left[\frac{W}{R} \left\{ \frac{1}{(E_b / I_0)} - \frac{1}{SNR} \right\} \right]$$

- Number of user is $K = 16$

$$K = 1 + \left[\frac{1.25 \times 10^6}{8.2 \times 10^3} \left\{ \frac{1}{(5)} - \frac{1}{10} \right\} \right] = 16.2$$

The received power from the i -th transmitter may be represented as

$$P_i = \frac{P_o}{d_i^\alpha}$$

Here P_o = received power at unit distance
 d_i = distance from the i -th transmitter to j -th transmitter
 α = propagation constant

The ratio of the power received from the i -th transmitter to that received from the j -th transmitter can be represented by

$$P_j = \left[\frac{d_i}{d_j} \right]^\alpha P_i$$

$$\left(\frac{E_b}{I_0}\right)_i = \frac{(W / R) P_i}{\sigma_n^2 + \sum_{j \neq i} \left[\frac{d_j}{d_i} \right]^\alpha P_j}$$

The equation can be solved if the distance term is obtained, here,

$$\sum_{j \neq i} \left[\frac{d_j}{d_i} \right]^\alpha \leq \frac{W}{R} \left[\frac{1}{E_b / I_0} - \frac{1}{(SNR)_i} \right]$$



... (Continued) Near-far effect...

- The distance term gives the near-far effect.
- If d_i is less than d_j , then fewer terms can be added until the sum becomes equal to the right-hand side.
- This results in smaller number of effective users

... (Continued) Example...

Assume, all transmitters are at the same distance from the receiver except for user 1, that is

$$d_i = d_1 \text{ where } i \text{ is the user } 1$$

$$d_1 = 0.5 d_j.$$

$$\alpha = 3.5 \text{ (propagation loss exponent)}$$

This gives us the number of users (for situation in example 2) as -

$$\left(\frac{E_b}{I_0}\right)_i = \frac{(W / R)P_i}{\sigma_n^2 + (K - 2)P_j + \left(\frac{d_j}{d_1}\right)^{3.5} P_j}$$

$$K = 2 - \left(\frac{d_j}{d_1} \right)^{3.5} + \left[\frac{1.25 \times 10^6}{8.2 \times 10^3} \left\{ \frac{1}{(5)} - \frac{1}{10} \right\} \right] = 5.86$$

- The number of users has been reduced by a factor of 3 (only 5 users) simply by virtue of one of the transmitters being 2 times closer than all of the others.
- The system would eventually fail as multiple access system since only one user could be supported and none of the others would be able to be received with the desired output SNR, if $d_1 \leq \frac{d_j}{2.78}$