# Sistem Komunikasi 1

# Bab 12
# Modulasi Digital & Kinerjanya
# BPSK, QPSK

# What is Modulation?

- Encoding information in a manner suitable for transmission.
  - Translate baseband source signal to bandpass signal
  - Bandpass signal: “modulated signal”

- How?
  - Vary amplitude, phase or frequency of a carrier

- Demodulation: extract baseband message from carrier

# Modulasi Analog

Persamaan sinyal pembawa /carrier :

$$Vc(t) = Vc \sin(\omega_c t + \theta)$$

**Modulasi amplitude**

**(amplitude modulation, AM)**

Modulasi sudut

(angle modulation)

$(\omega_c t + \theta)$

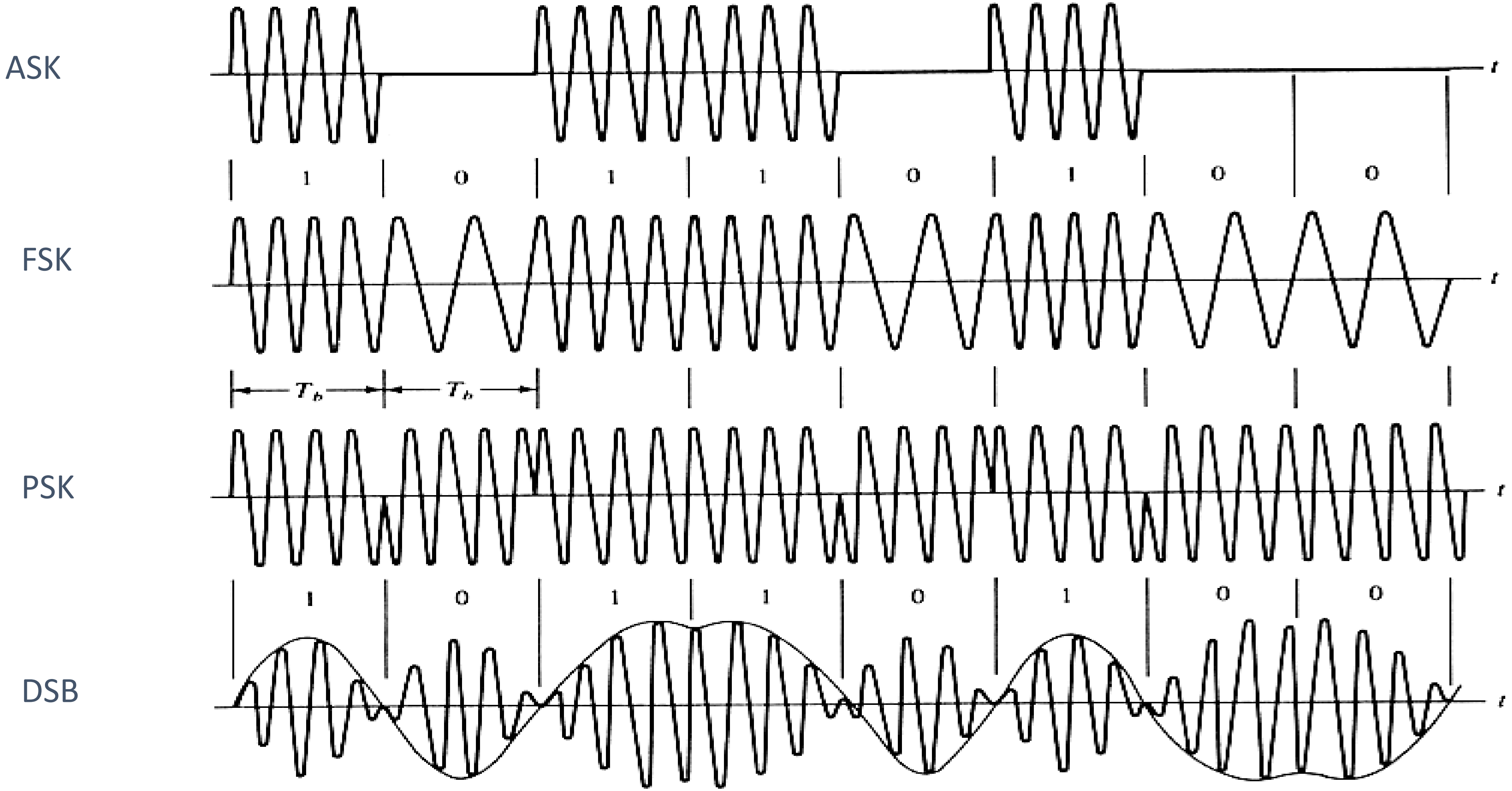
Modulasi frekuensi

(frequency modulation, FM)

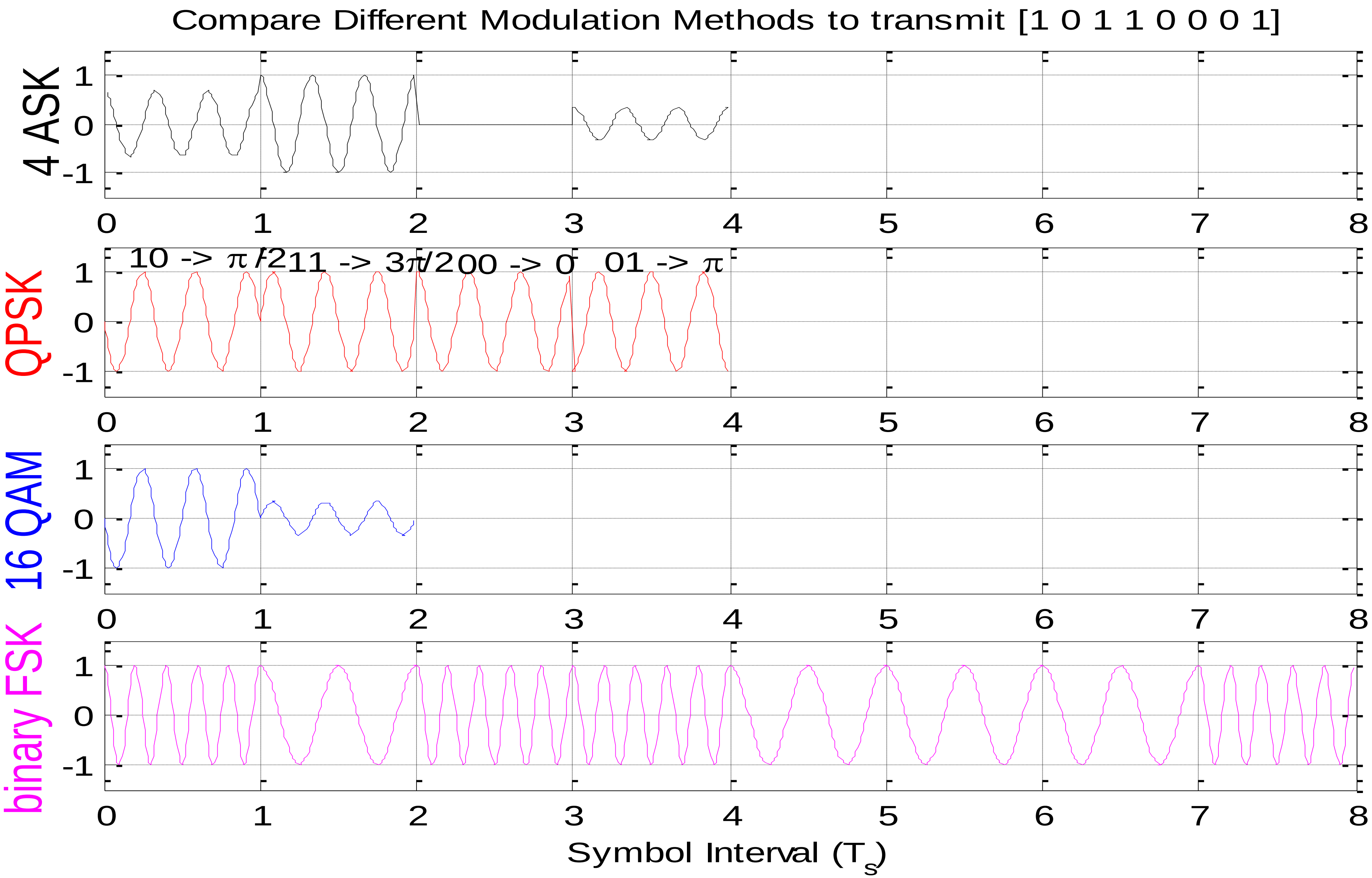
Modulasi fase

(phase modulation, PhM)

# Gambar beberapa modulasi Digital



# Gambar lain beberapa modulasi Digital



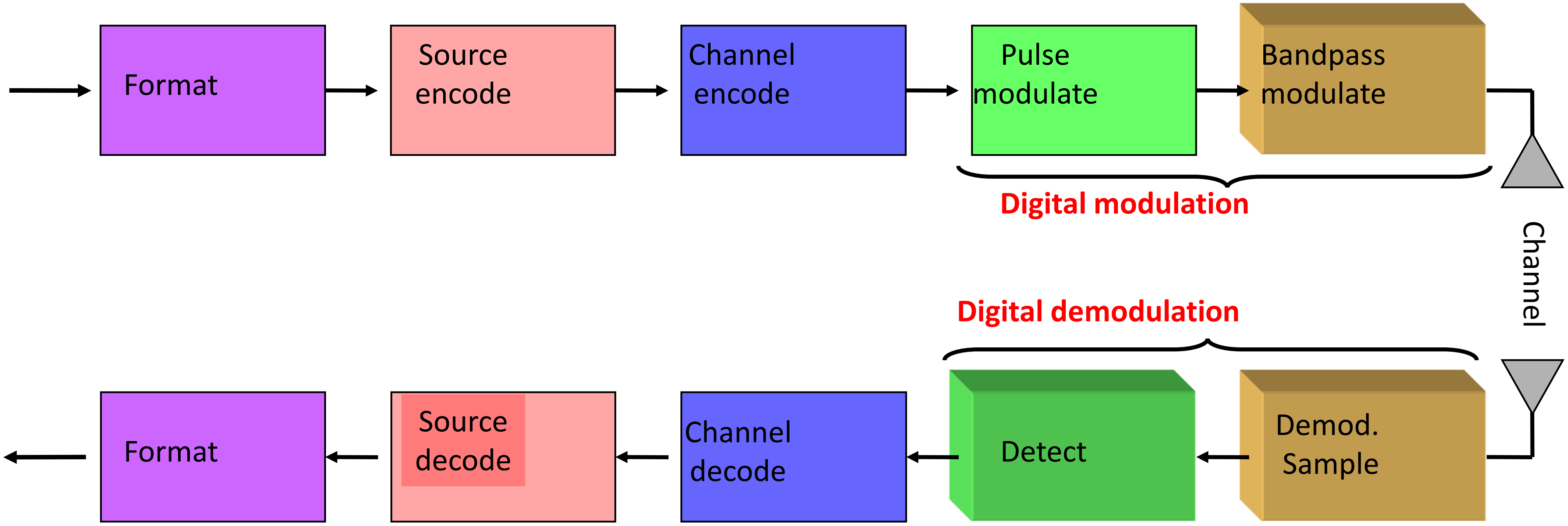
# Digital vs Analog Modulation

- Cheaper, faster, more power efficient
- Higher data rates, power error correction, impairment resistance:
  - Using coding, modulation, diversity
  - Equalization, multicarrier techniques for ISI mitigation
- More efficient multiple access strategies, better security: CDMA, encryption etc

# Goals of Modulation Techniques

- **High Bit Rate**
- **High Spectral Efficiency** (*max Bps/Hz*)
- **High Power Efficiency** (*min power to achieve a target BER*)
- **Low-Cost/Low-Power Implementation**
- **Robustness to Impairments**

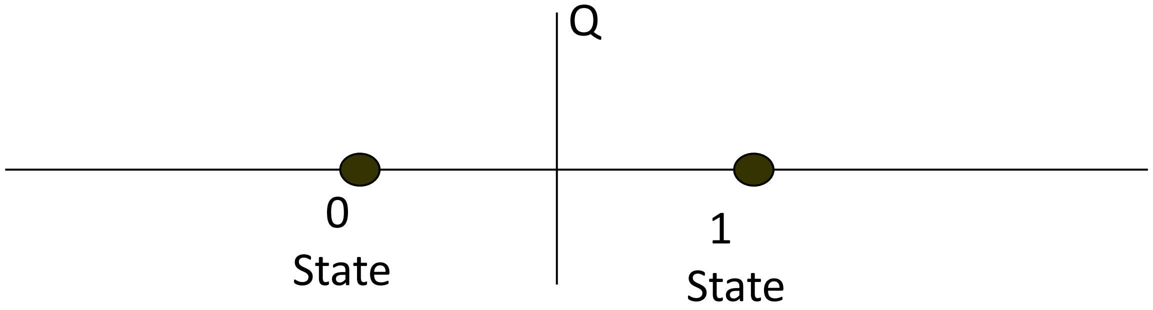
# Block diagram of a Digital Communication System



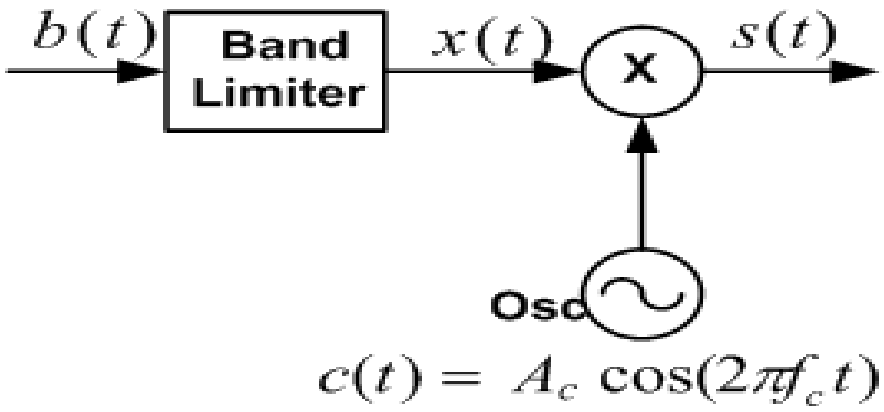
# Binary Phase Shift Keying

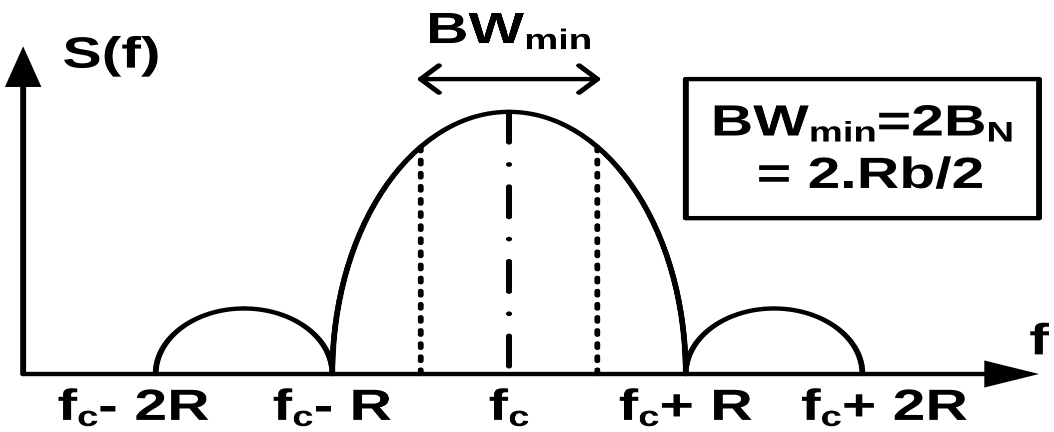
- Menggunakan alternatif-alternatif fasa gelombang sinus utk mengkodekan bit-bit:
  - Fasa dipisahkan 180 derajat.
  - Sederhana utk diimplementasikan, tidak efisien dalam penggunaan bandwidth.
  - Sangat kokoh, sering digunakan secara extensif pada komunikasi satelit.

$$\boxed{\begin{array}{ll} s_1(t) = A_c \cos(2\pi f_c t) & \text{binary '1'} \\ s_2(t) = A_c \cos(2\pi f_c t + \pi) & \text{binary '0'} \end{array}}$$



# Pembangkitan BPSK

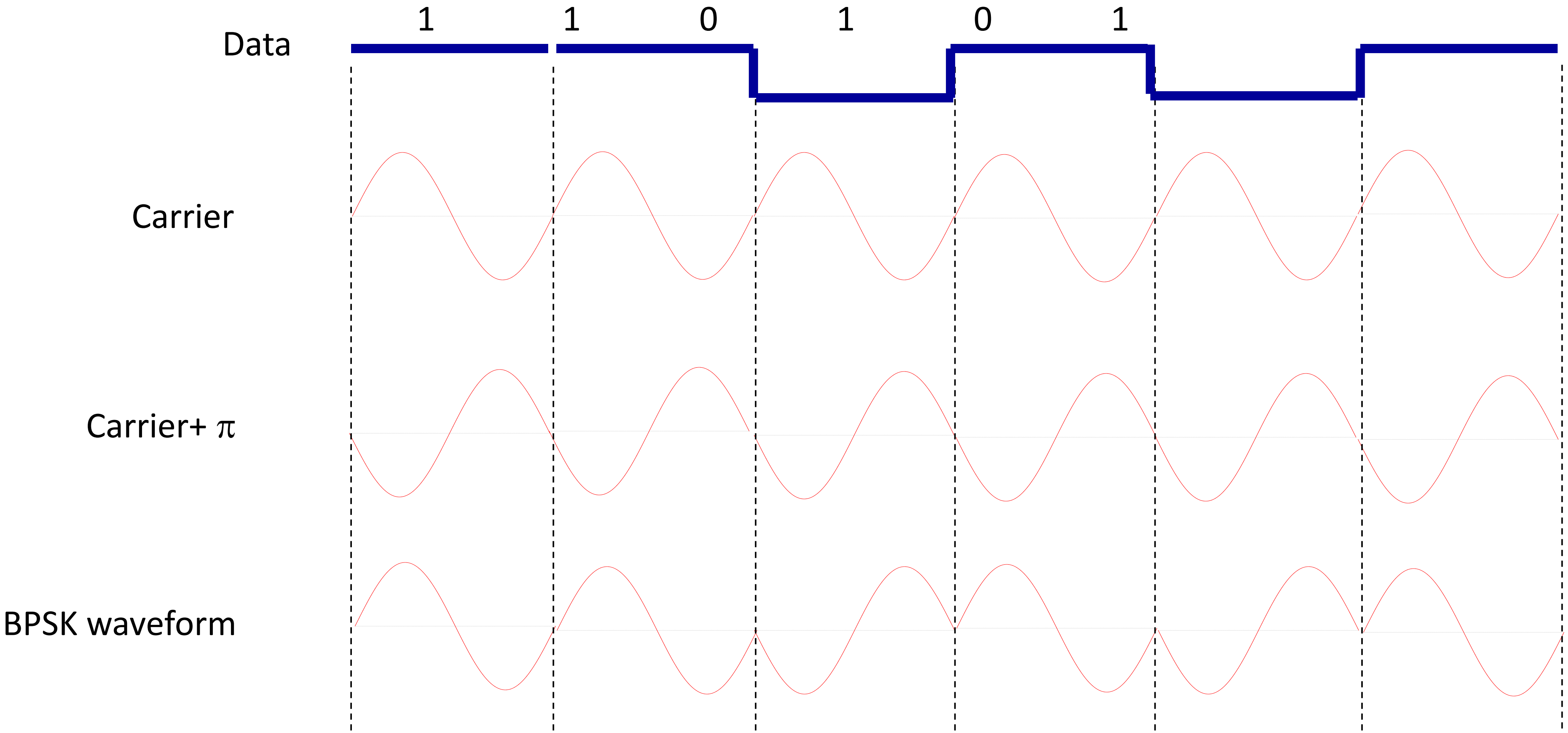




$B_N$=Bandwidth Nyquist

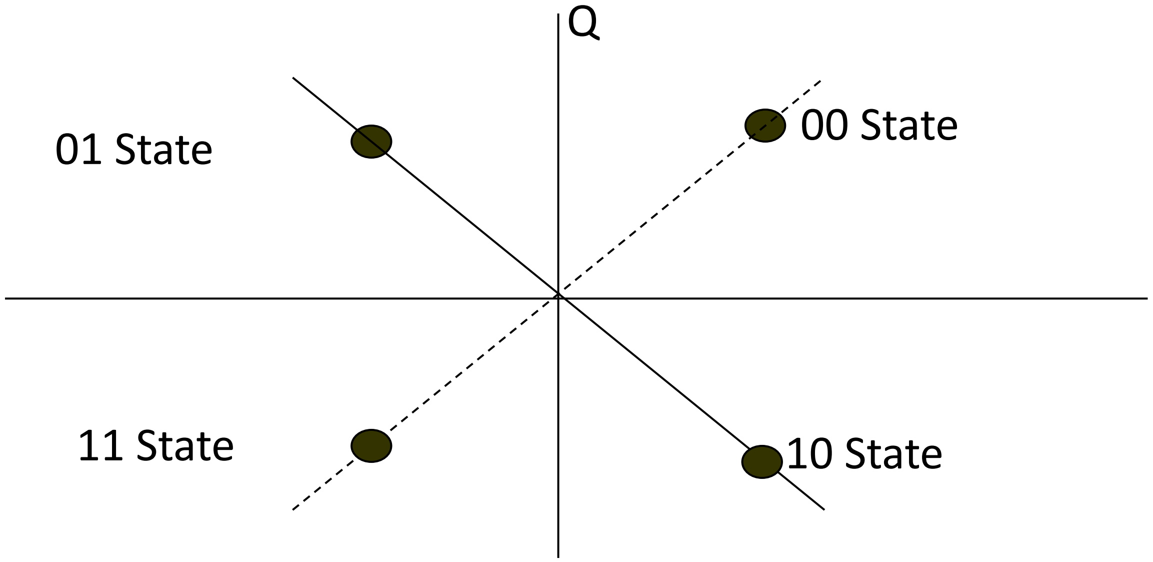
$$s(t) = \begin{cases} s_1(t) = A_c \cos(2\pi f_c t) & \text{binary '1'} \\ s_2(t) = A_c \cos(2\pi f_c t + \pi) & \text{binary '0'} \end{cases}$$

# Contoh BPSK



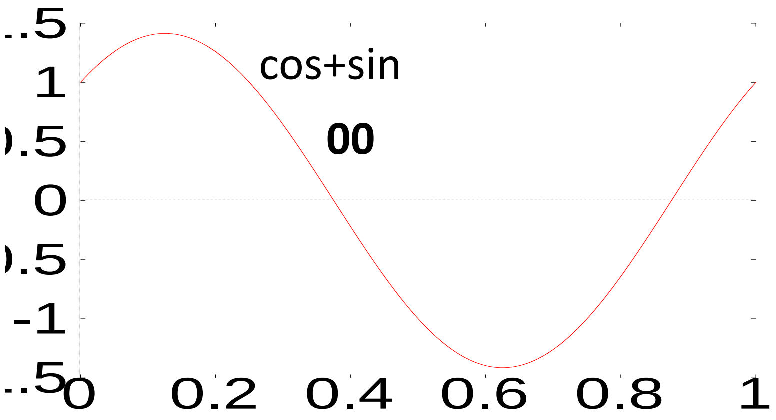
# Quadrature Phase Shift Keying

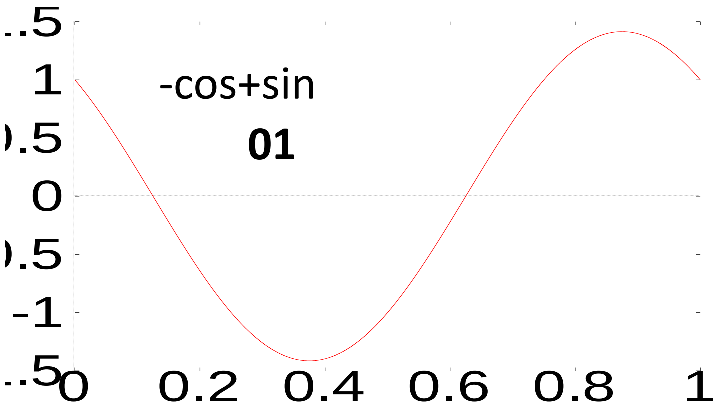
- Teknik modulasi multilevel : 2 bit per symbol
- Lebih efisien spektrum, lebih kompleks receiver.
- Dua kali lebih efisien bandwidth daripada BPSK

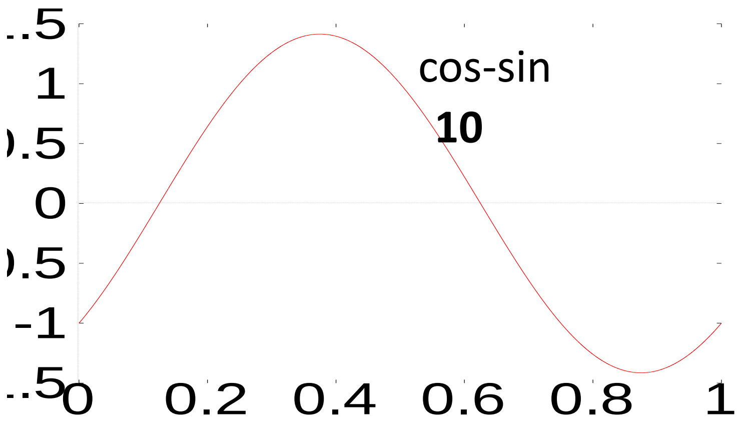


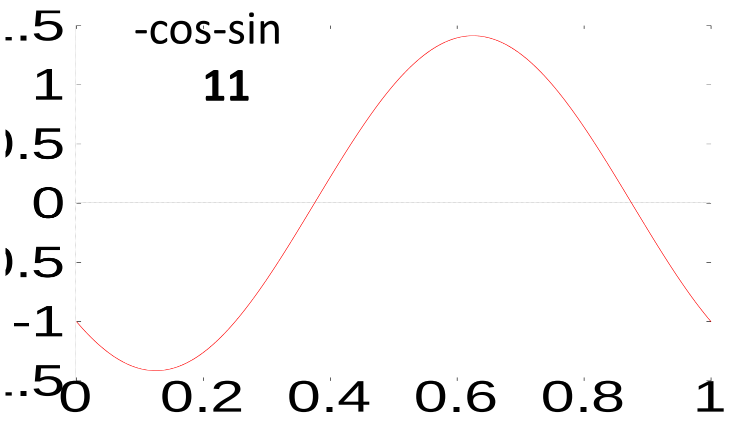
Phase of Carrier: π/4, 2π/4, 5π/4, 7π/4

# 4 bentuk gelombang berbeda:

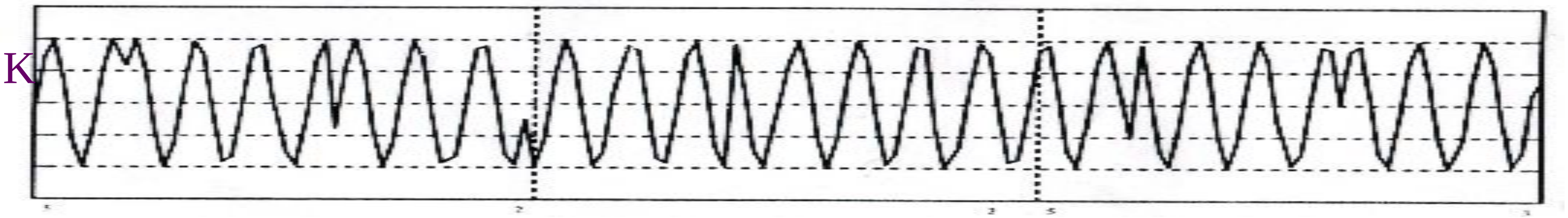




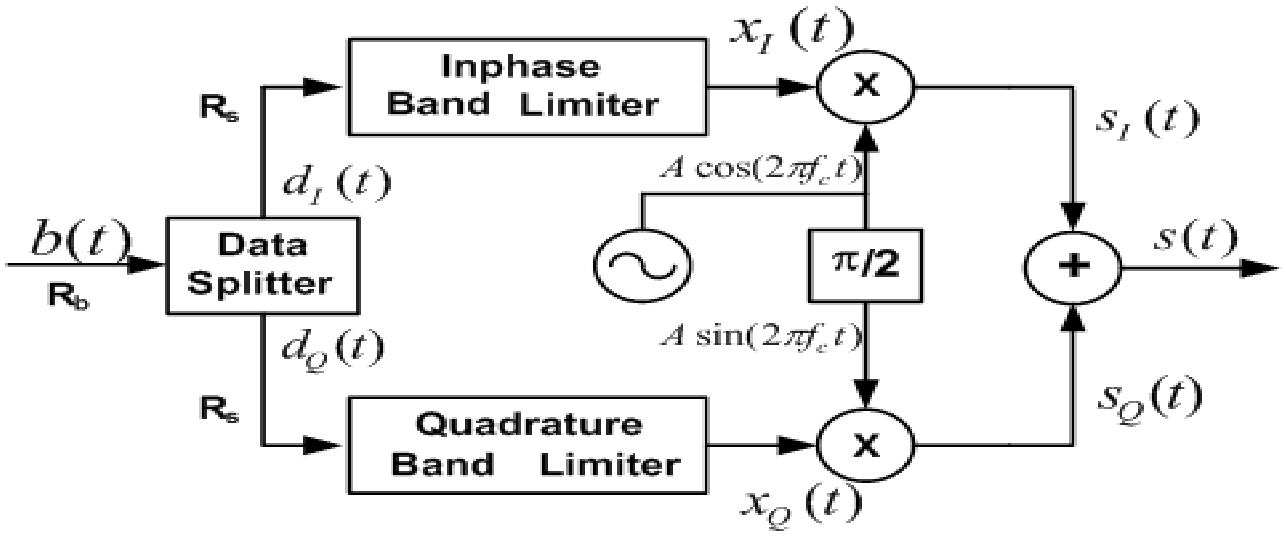


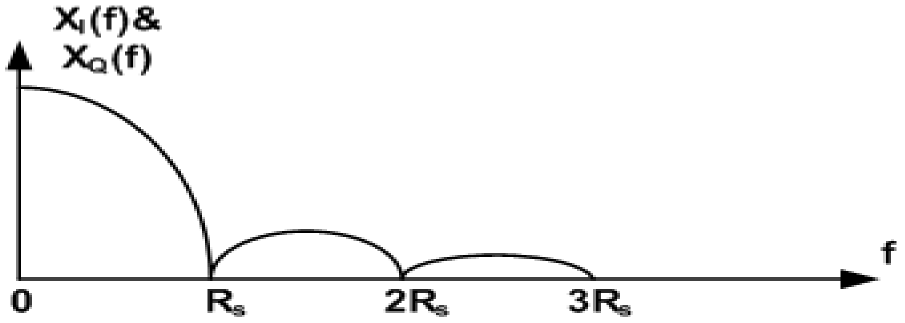


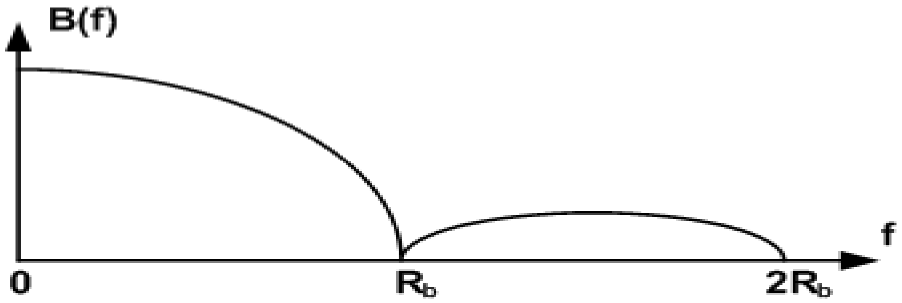
Bentuk Sinyal QPSK

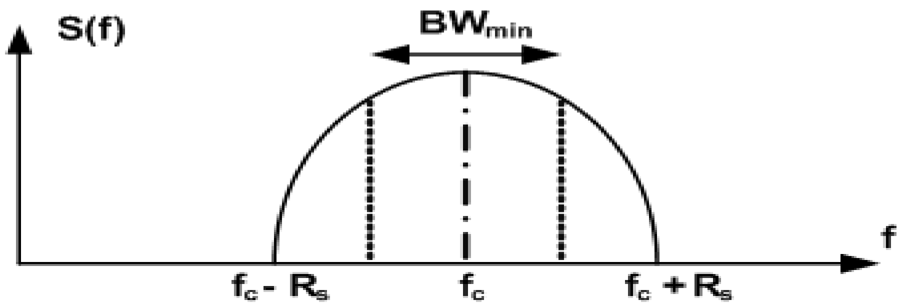


# Pembangkitan sinyal QPSK

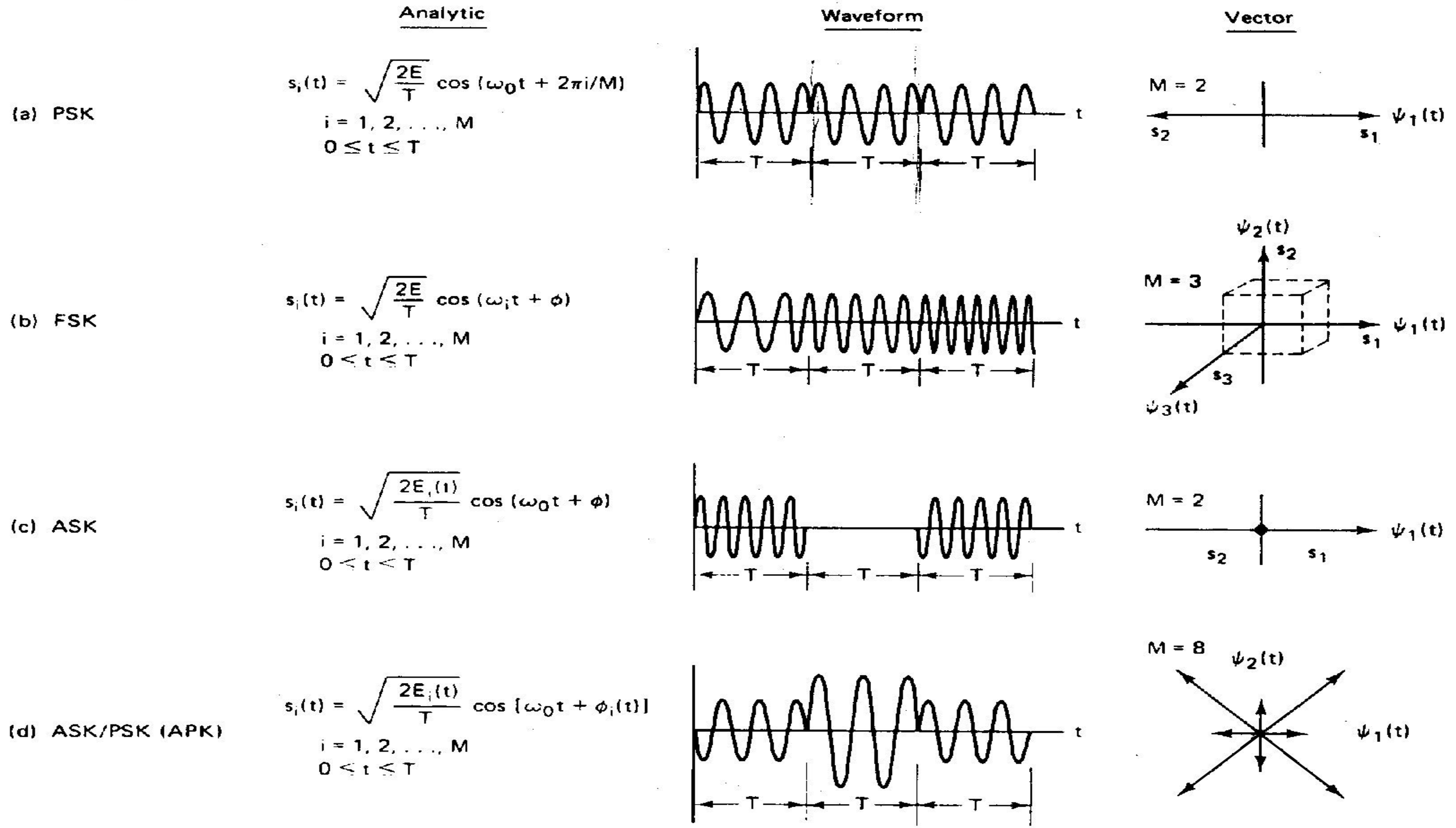








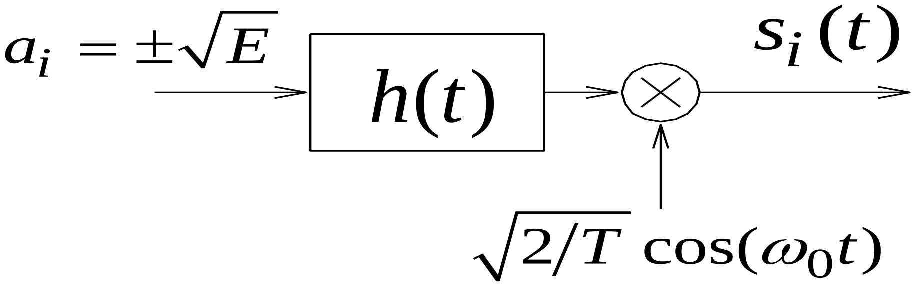
# Signal Space of several modulation



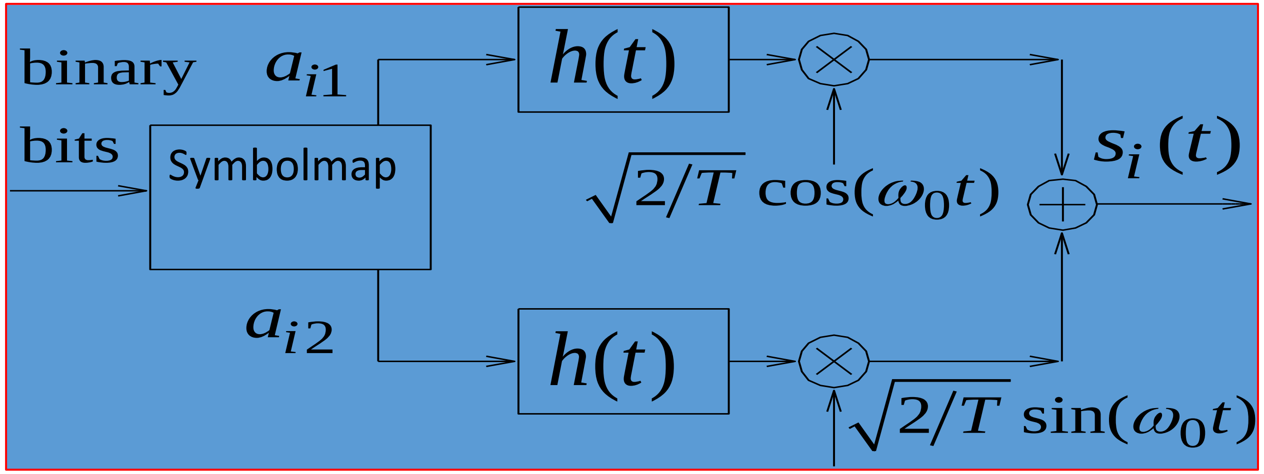
| | Analytic | Waveform | Vector |
|---|---|---|---|
| (a) PSK | $s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + 2\pi i/M)$<br>$i = 1, 2, \ldots, M$<br>$0 \le t \le T$ | | M = 2 |
| (b) FSK | $s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi)$<br>$i = 1, 2, \ldots, M$<br>$0 \le t \le T$ | | M = 3 |
| (c) ASK | $s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos(\omega_0 t + \phi)$<br>$i = 1, 2, \ldots, M$<br>$0 \le t \le T$ | | M = 2 |
| (d) ASK/PSK (APK) | $s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos[\omega_0 t + \phi_i(t)]$<br>$i = 1, 2, \ldots, M$<br>$0 \le t \le T$ | | M = 8 |

# PSK modulator

- Special case: BPSK modulator



- General case: M-ary PSK modulator



Note:

Inputs are signal-space vector.

Carriers are in basis form.

$$s_i(t) = a_{i1}\sqrt{2/T}\cos(\omega_0 t) + a_{i2}\sqrt{2/T}\sin(\omega_0 t)$$

$$\mathbf{s}_i = (a_{i1}, a_{i2}) = \left(\sqrt{E}\cos(2\pi i/M),\ -\sqrt{E}\sin(2\pi i/M)\right)$$

# Bandwidth of PSK signal waveform

- Just like DSB modulation:

$$\boxed{W_{\text{PSK}} = 2W_{\text{baseband}}}$$

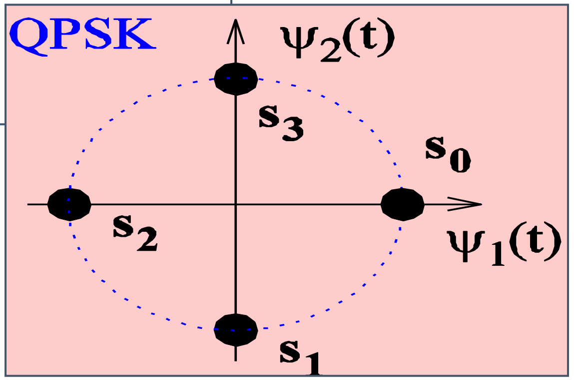
- **Exercise :** Consider QPSK transmission with date rate 2000 bps. The magnitude of the signal si(t) is √2E/T =1 volt.
  - a) What is the minimum PSK signal bandwidth?
  - b) Find the signal space points
  - c) Draw the constellation
  - d) Find signal waveform for transmitting {1001}.

a) $R_s = R_b/(\log_2 M) = 2000/2 = 1000.\ \ W_{\text{PSK}} = 2W_{\text{baseband,min}} = 2R_s/2 = 1000\text{Hz}.$

b) $\mathbf{s}_i = (\sqrt{E}\cos 2\pi i/4, -\sqrt{E}\sin 2\pi i/4),\ \text{where}\ \ E = T/2 = 0.5\times 10^{-3},\quad i = 1,\cdots,4$

d) Define mapping as: $\{00{:}0,\ 01{:}\pi,\ 10{:}\pi/2,\ 11{:}3\pi/2\}.$

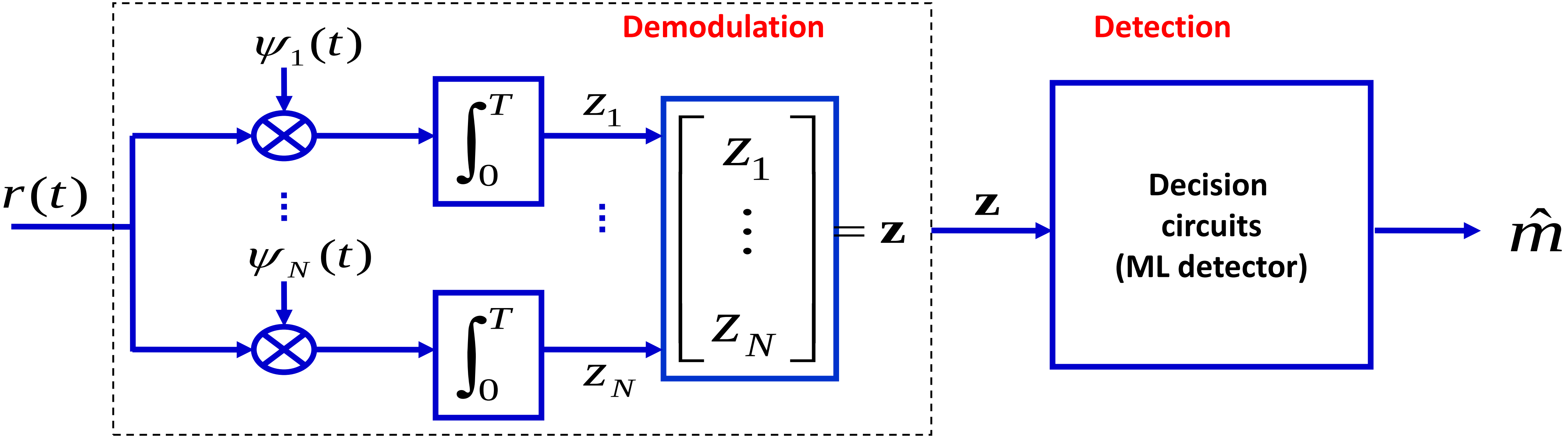
Then $\{10\} \rightarrow s_1(t) = \cos(\omega_0 t + \pi/2).\quad \{01\} \rightarrow s_2(t) = \cos(\omega_0 t + \pi)$



Phase $\phi_i(t)$ in $s_i(t)$ is different from phase of $\mathbf{s}_i$ (phase in signal space)

# Demodulation and detection

- **Demodulation:** The receiver signal is converted to baseband, filtered and sampled.
- **Detection:** Sampled values are used for detection using a decision rule such as ML detection rule.



# Demodulations type:

- Some notations
  - Carrier: $s(t) = A(t)\cos[\omega_0 t + \phi(t)], \qquad \omega_0 = 2\pi f_0$
  - Modulation types with respect to carrier parameters

| Modulation | Varying parameter | Demodulation |
|---|---|---|
| **PSK** | $\phi(t)$ | **Coherent or noncoherent** |
| **QAM** | both $A(t)$ and $\phi(t)$ | **Coherent or noncoherent** |
| **FSK** | $\omega_0$ | **Coherent or Noncoherent** |

# Demodulations type:

- **Coherent detection / synchronous detection**
  - Receiver exploits knowledge of carrier's phase to detect signals
  - Require accurate phase (and frequency as well) estimation
  - Higher performance (lower error rate), but increased complexity
  - Extremely similar to baseband processing mathematically if signal space is used

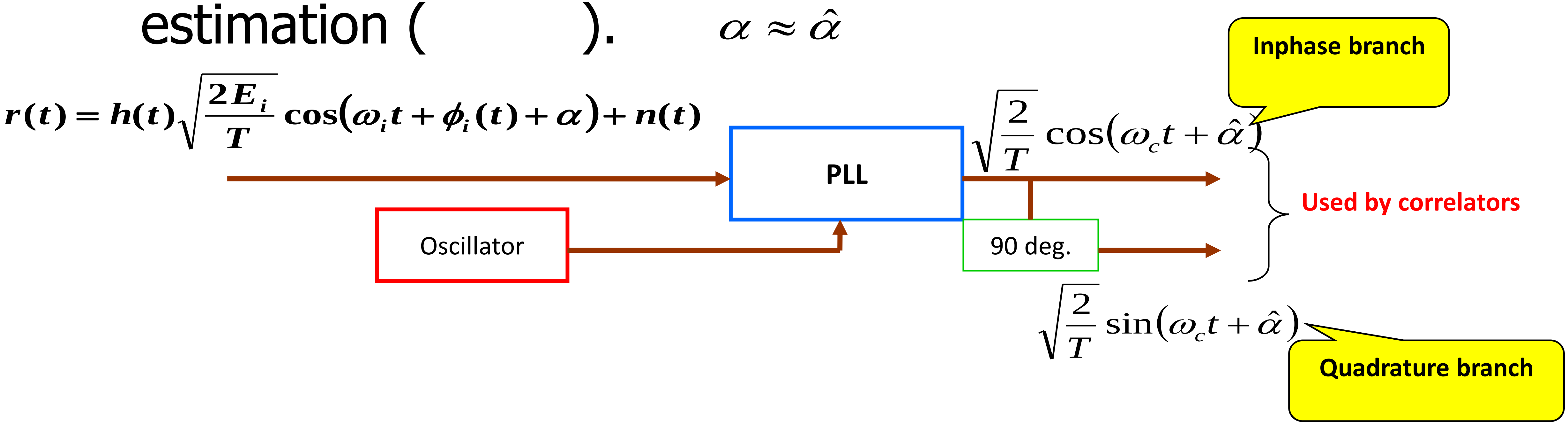
- **Noncoherent detection / asynchronous detection**
  - Receiver does not exploit carrier phase
  - Do not need accurate phase estimation
  - Reduced complexity, but lower performance (higher error rate)
  - Unique for bandpass processing: via differential encoding, or FSK energy detector

# Coherent detections

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Source of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.

# Coherent detection ..

- Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ( $\alpha \approx \hat{\alpha}$ ).



$$r(t) = h(t)\sqrt{\frac{2E_i}{T}}\cos\left(\omega_i t + \phi_i(t) + \alpha\right) + n(t)$$

$$\sqrt{\frac{2}{T}}\cos\left(\omega_c t + \hat{\alpha}\right)$$

**Inphase branch**

$$\sqrt{\frac{2}{T}}\sin\left(\omega_c t + \hat{\alpha}\right)$$

**Quadrature branch**

**Used by correlators**

# Two dimensional modulation, demodulation and detection (M-PSK)

- M-ary Phase Shift Keying (M-PSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(\omega_c t + \frac{2\pi i}{M}\right)$$

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \ldots, M$$

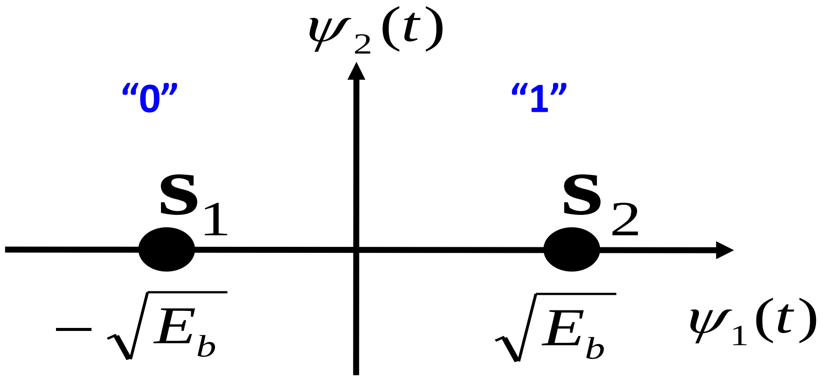
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \qquad \psi_2(t) = -\sqrt{\frac{2}{T}} \sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s} \cos\left(\frac{2\pi i}{M}\right) \qquad a_{i2} = \sqrt{E_s} \sin\left(\frac{2\pi i}{M}\right)$$

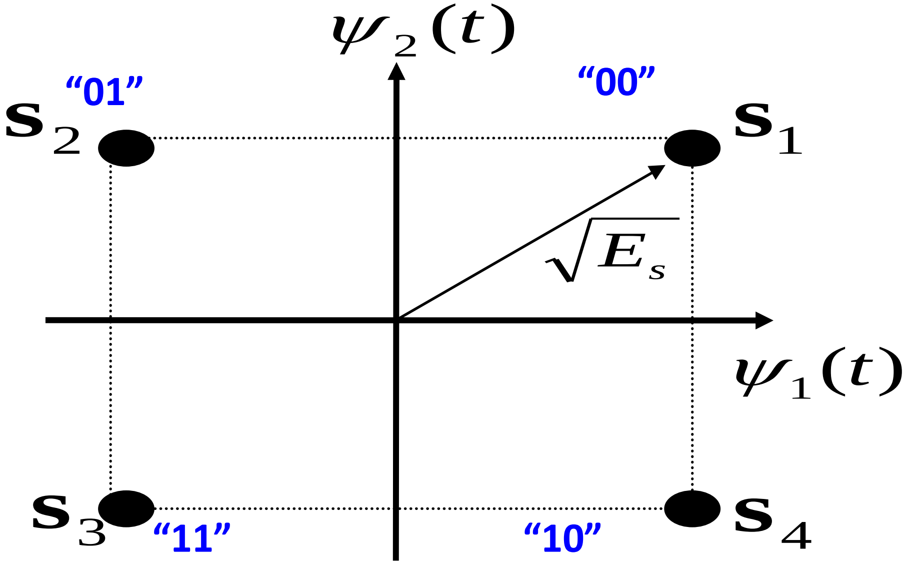
$$E_s = E_i = \left\| \mathbf{s}_i \right\|^2$$

# Two dimensional mod… (MPSK)

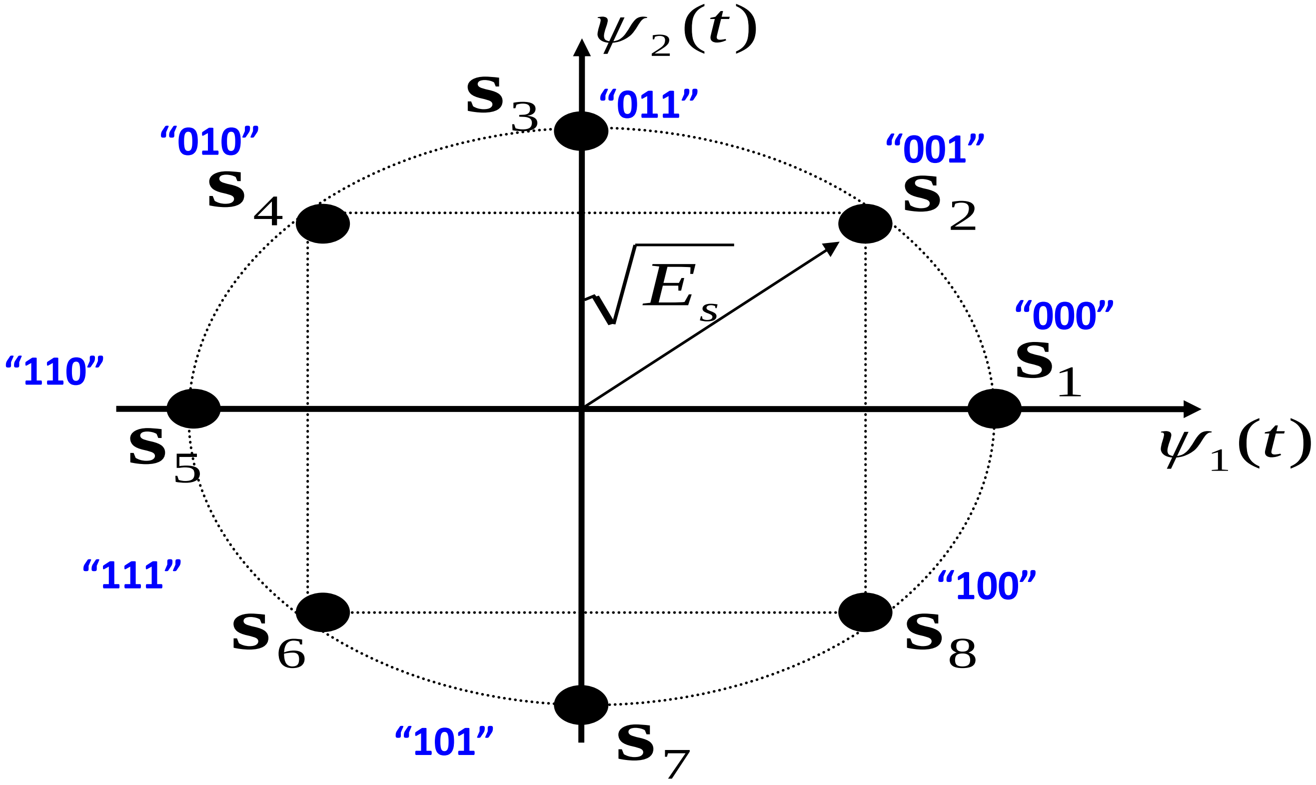
**BPSK (M=2)**



**QPSK (M=4)**

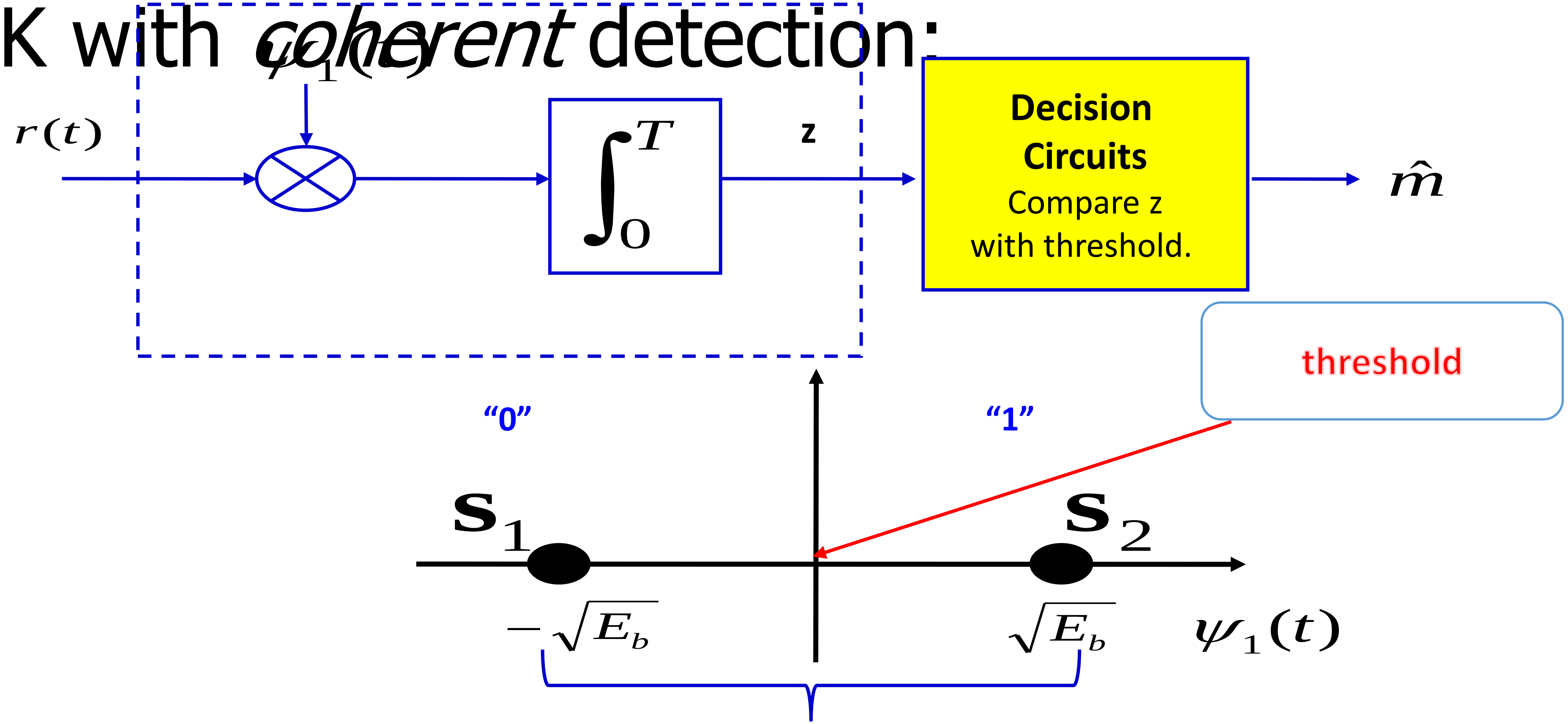


**8PSK (M=8)**



# Demodulation BPSK

- BPSK with *coherent* detection:



**Decision Circuits**

Compare z with threshold.

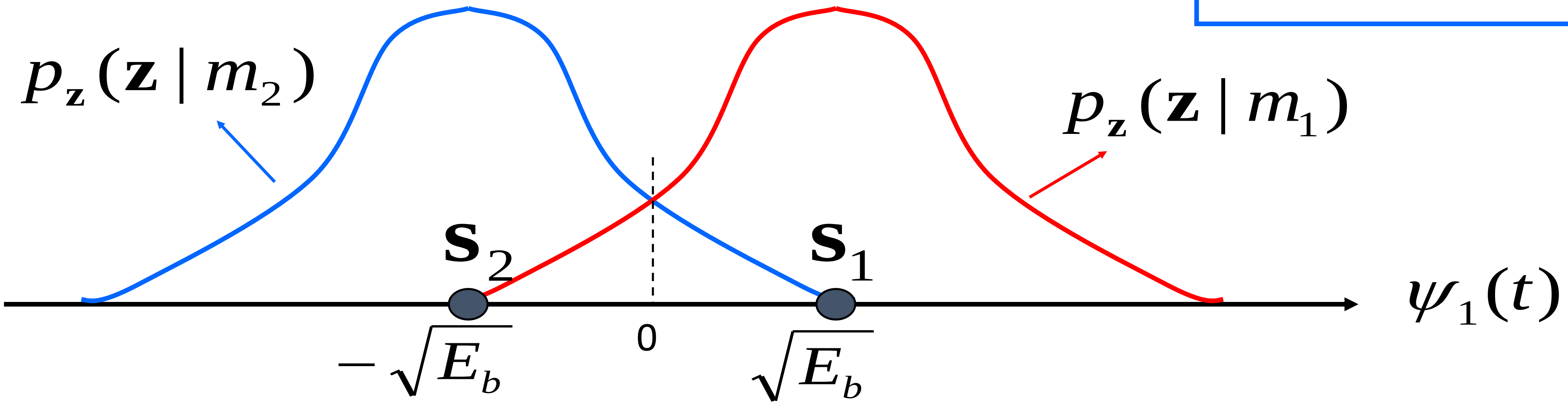
$$\|\mathbf{s}_1 - \mathbf{s}_2\| = 2\sqrt{E_b}$$

# Error probability …

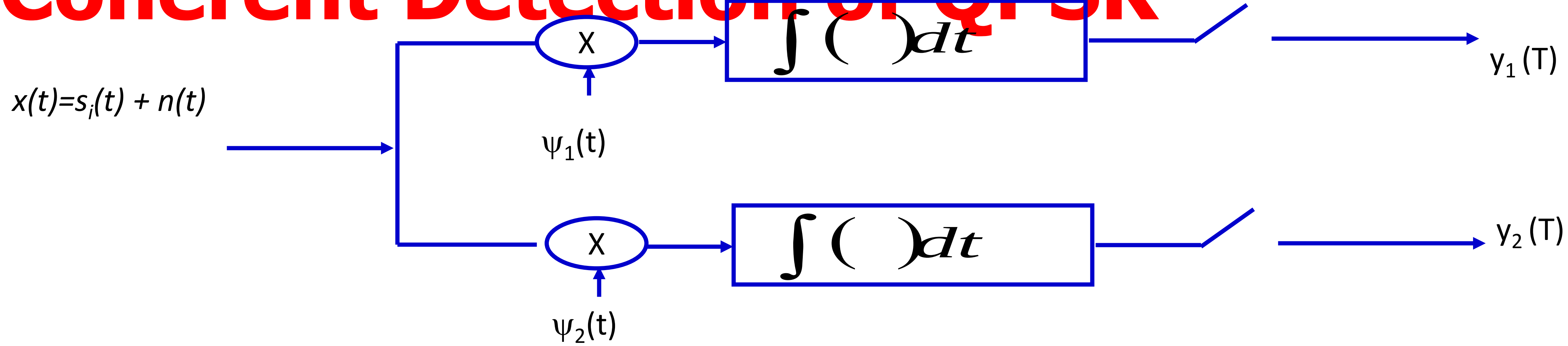
- BPSK with *coherent* detection (with perfect carrier synchronization):

$$P_B = Q\left(\frac{\|\mathbf{s}_1 - \mathbf{s}_2\| / 2}{\sqrt{N_0 / 2}}\right)$$

$$\boxed{P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$



# Coherent Detection of QPSK



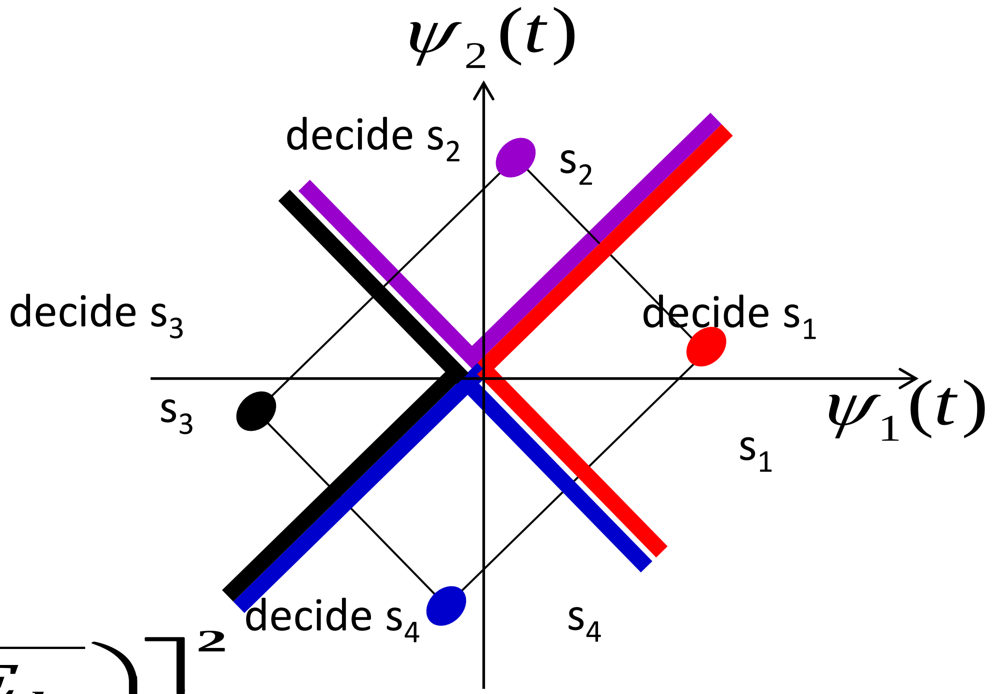
$$y_1(T) = \sqrt{E_S}\cos\left[(2i-1)\frac{\pi}{4}\right] + n_1 = \pm\sqrt{\frac{E_S}{2}} + n_1$$

$$y_2(T) = \sqrt{E_S}\sin\left[(2i-1)\frac{\pi}{4}\right] + n_2 = \mp\sqrt{\frac{E_S}{2}} + n_2$$

**QPSK can be seen as two binary PSK acting independently.**

# Demodulation M-PSK

- Coherent detection of Q-PSK



Decision Region QPSK

$$p_C = (1 - p_{BPSK-I})^2 = \left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]^2$$

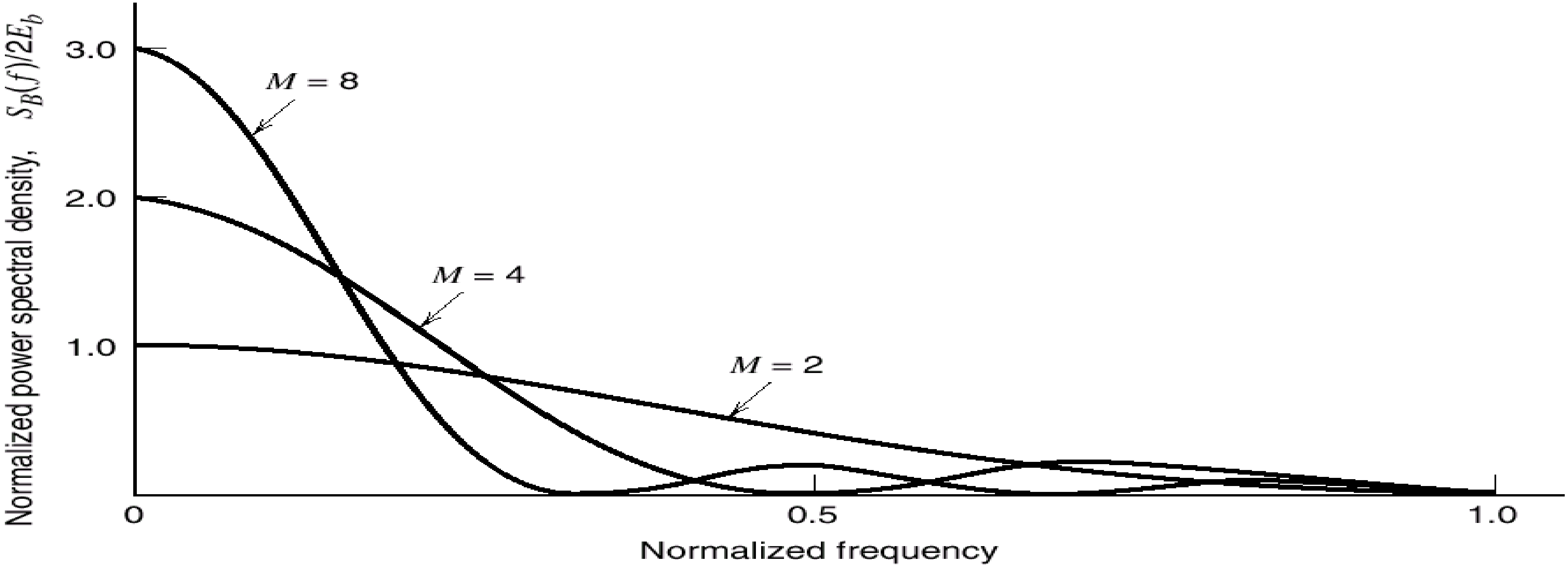
$$p_e = 1 - p_c = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

$$p_e \approx Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

# Power Spectra of M-Ary PSK

$$S_B(f) = 2E \sin c^2(Tf)$$

$$S_B(f) = 2E_b \log M \sin c^2(T_b f \log_2 M)$$



# QPSK vs. BPSK

- Let’s compare the two based on BER and bandwidth

| BER | | Bandwidth | |
|---|---|---|---|
| **BPSK** | **QPSK** | **BPSK** | **QPSK** |
| $P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ | $P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ | **$R_b$** | **$R_b/2$** |

$$\underbrace{P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \qquad P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}_{\text{EQUAL}}$$

# Non-coherent detection

- Non-coherent detection:
  - No need in a reference in phase with the received carrier
  - Less complexity as compared to coherent detection at the price of higher error rate.

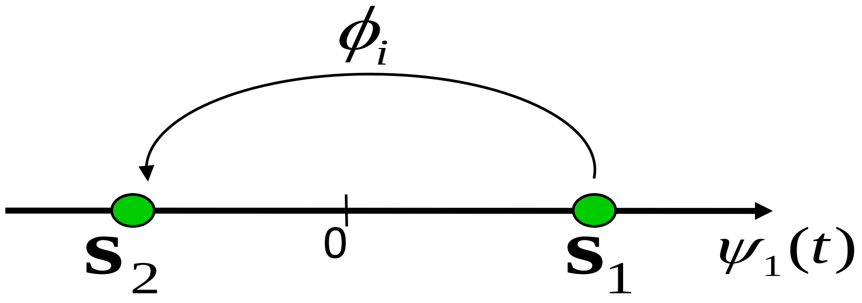
# Differential PSK…

- Differential encoding of the message
  - The symbol phase changes if the current bit is different from the previous bit.

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left(\omega_0 t + \theta_i(t)\right), \quad 0 \le t \le T, \quad i = 1,\dots,M$$

$$\boldsymbol{c(k) = \overline{c(k-1) \oplus m(k)} = c(k-1) \otimes m(k)}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Symbol index: | $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Data bits: | $\boldsymbol{m(k)}$ | | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| Diff. encoded bits: | $\boldsymbol{c(k)}$ | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| Symbol phase: | $\theta_k$ | $\pi$ | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | $\pi$ | $\pi$ |



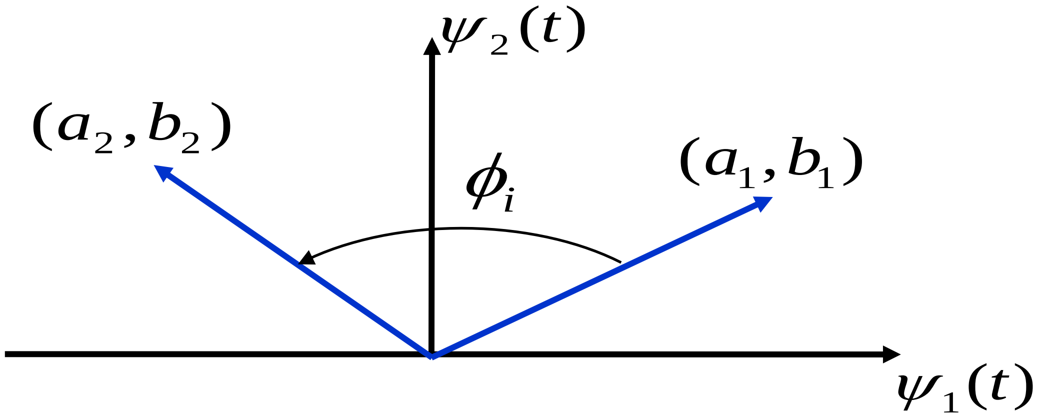
$$\theta_k(nT) = \theta_k((n-1)T) + \phi_i(nT)$$

# Coherent detection for diff encoded mod.

- assumes slow variation in carrier-phase mismatch during two symbol intervals.
- correlates the received signal with basis functions
- uses the phase difference between the current received vector and previously estimated symbol

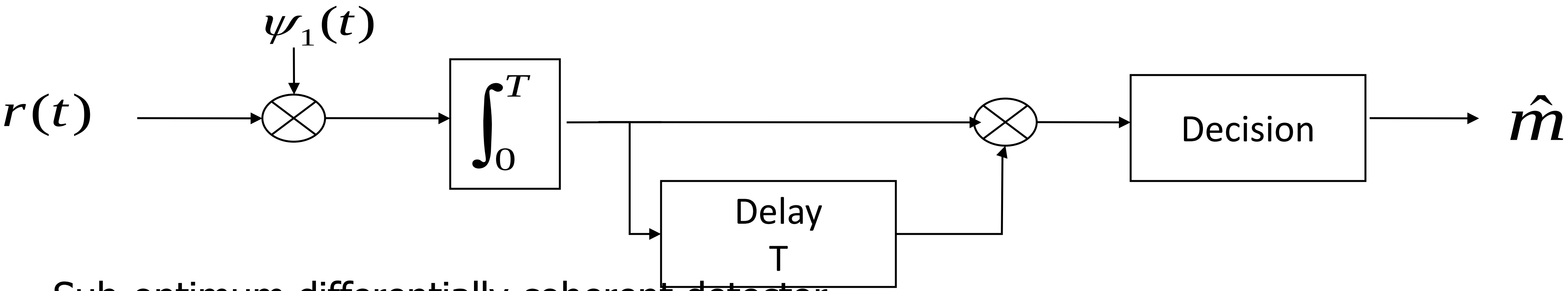
$$r(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_0 t + \theta_i(t) + \alpha\right) + n(t), \quad 0 \le t \le T$$

$$\left(\theta_i(nT) + \alpha\right) - \left(\theta_j((n-1)T) + \alpha\right) = \theta_i(nT) - \theta_j((n-1)T) = \phi_i(nT)$$

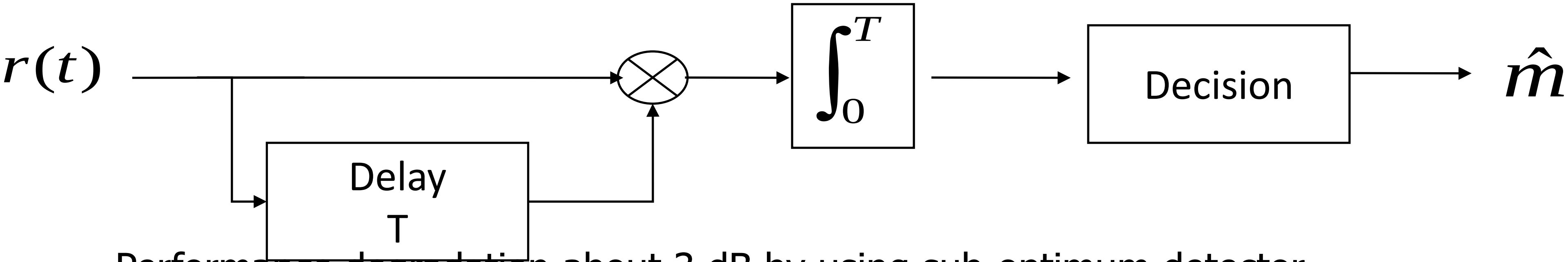


# DPSK detection …

- Optimum differentially coherent detector



- Sub-optimum differentially coherent detector



  - Performance degradation about 3 dB by using sub-optimum detector

# Non-Coherent Detection of Binary PSK- Differential PSK (DPSK)

The transmitted signal:

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_i) \qquad i = 0,1$$

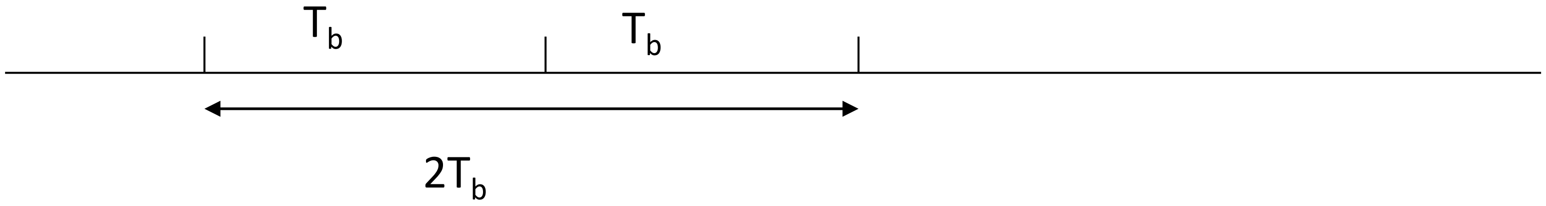
$$0 \le t \le T_b$$

The received signal:

$\alpha$ is assumed to change slowly reletive to consecutive symbols

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_i \underleftarrow{+ \alpha}) + n(t) \qquad i = 0,1$$

$$0 \le t \le T_b$$



$$[\theta_i(T_2) + \alpha] - [\theta_i(T_1) + \alpha] = 0 \qquad \longleftarrow \text{Equal phases}$$

$$[\theta_i(T_2) + \alpha] - [\theta_j(T_1) + \alpha] = \pi \qquad \longleftarrow \text{different phases}$$

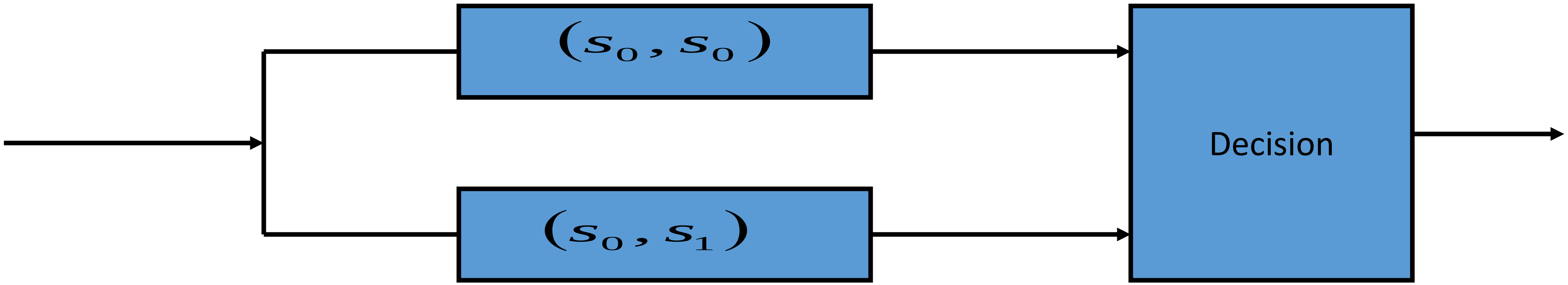
# Non-Coherent Differential PSK (DPSK)

Effectively in DPSK signaling we are transmitting each bit with the binary signaling pair:

$$(s_0, s_1)$$
$$(s_1, s_0)$$
$$(s_1, s_1)$$
$$(s_0, s_0)$$

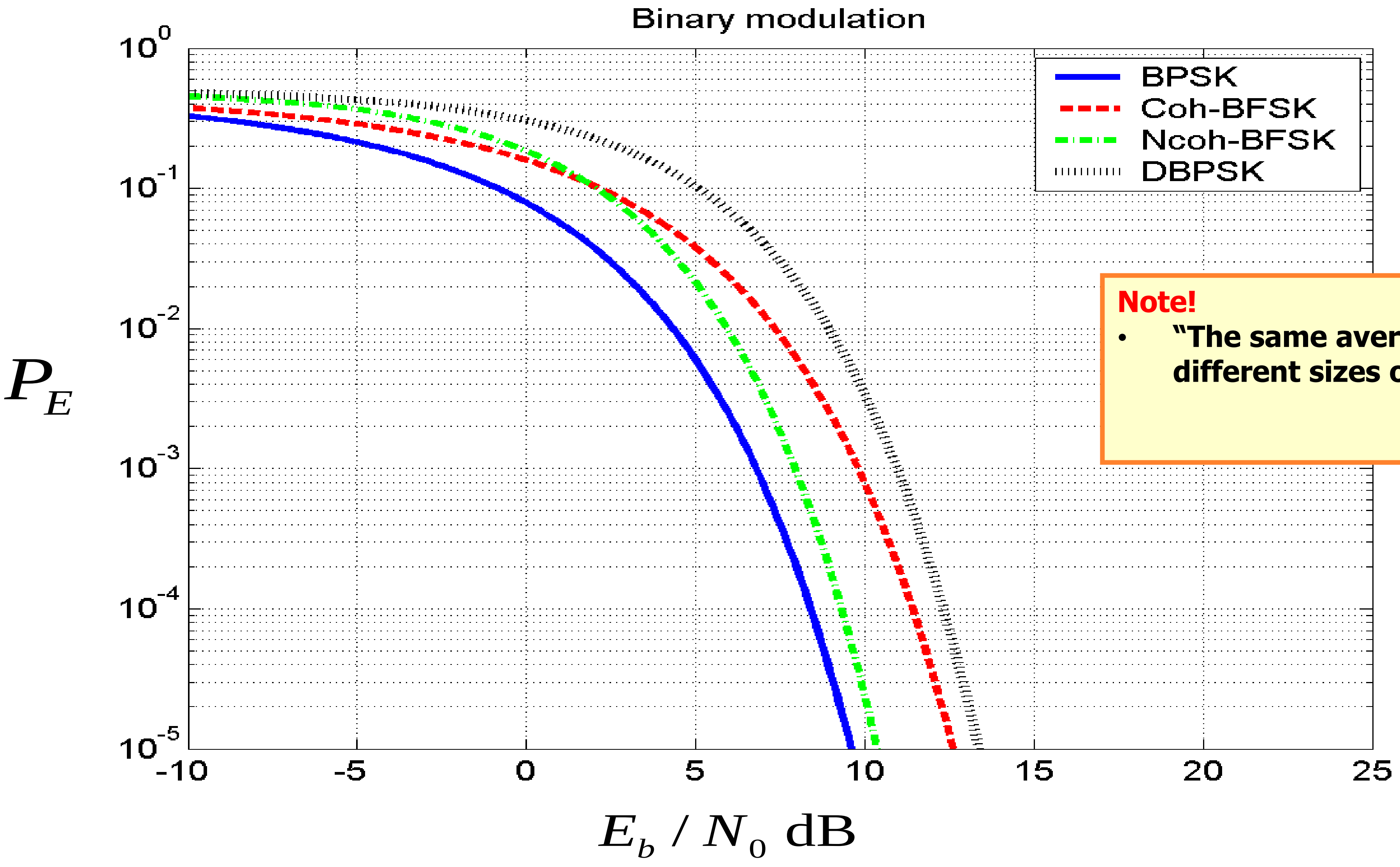
$$0 \le t \le 2T_b$$

Filters matched to signal envelope



$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

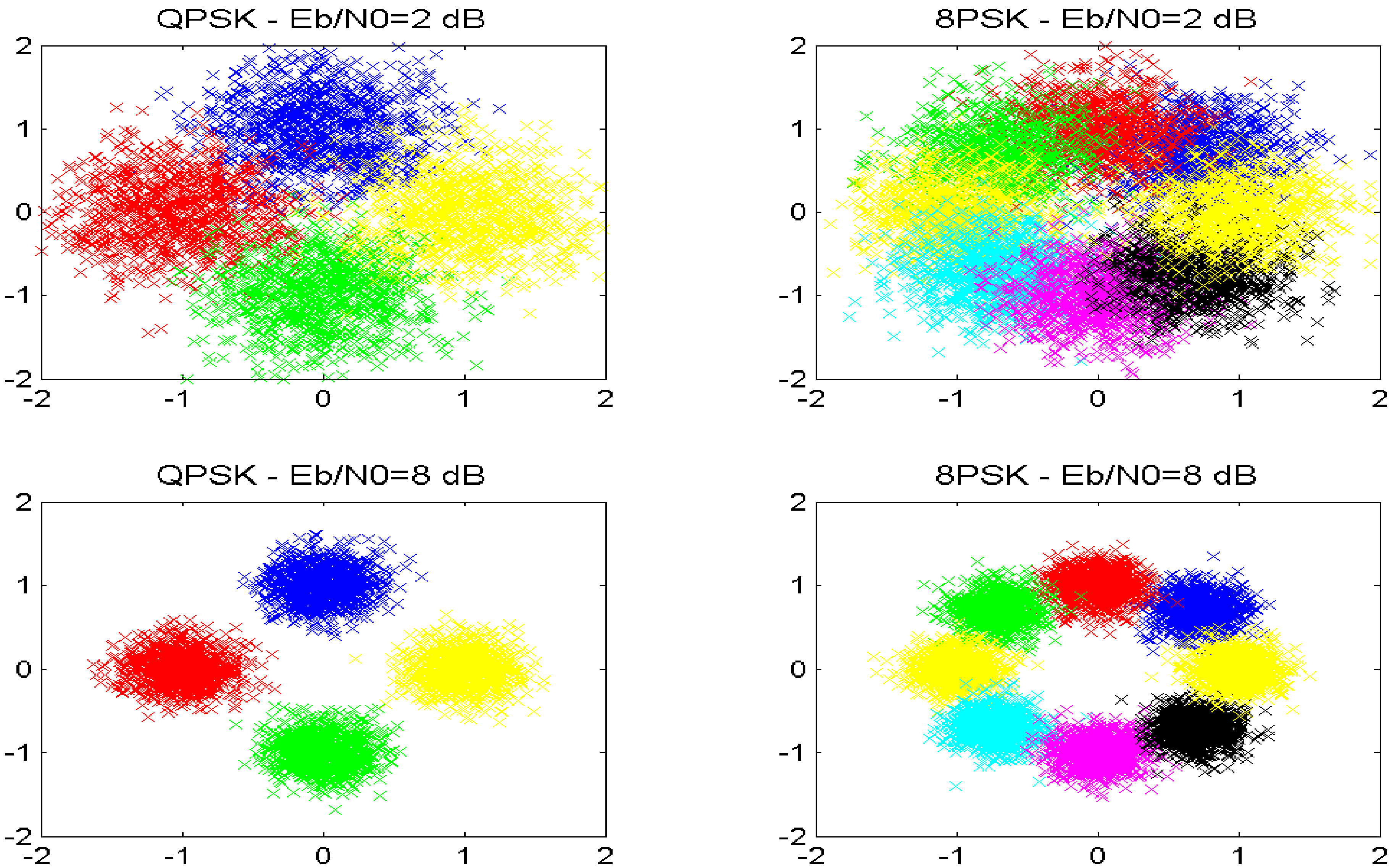
# Probability of symbol error for binary modulation



**Note!**

- **"The same average symbol energy for different sizes of signal space"**

# Example of samples of matched filter output for some bandpass modulation schemes



# End of Module 12