SISTEM TRANSPORTASI DAN DISTRIBUSI BARANG

Model Matematis dan Algoritma Heuristik Permasalahan Perutean Kendaran (VRP)

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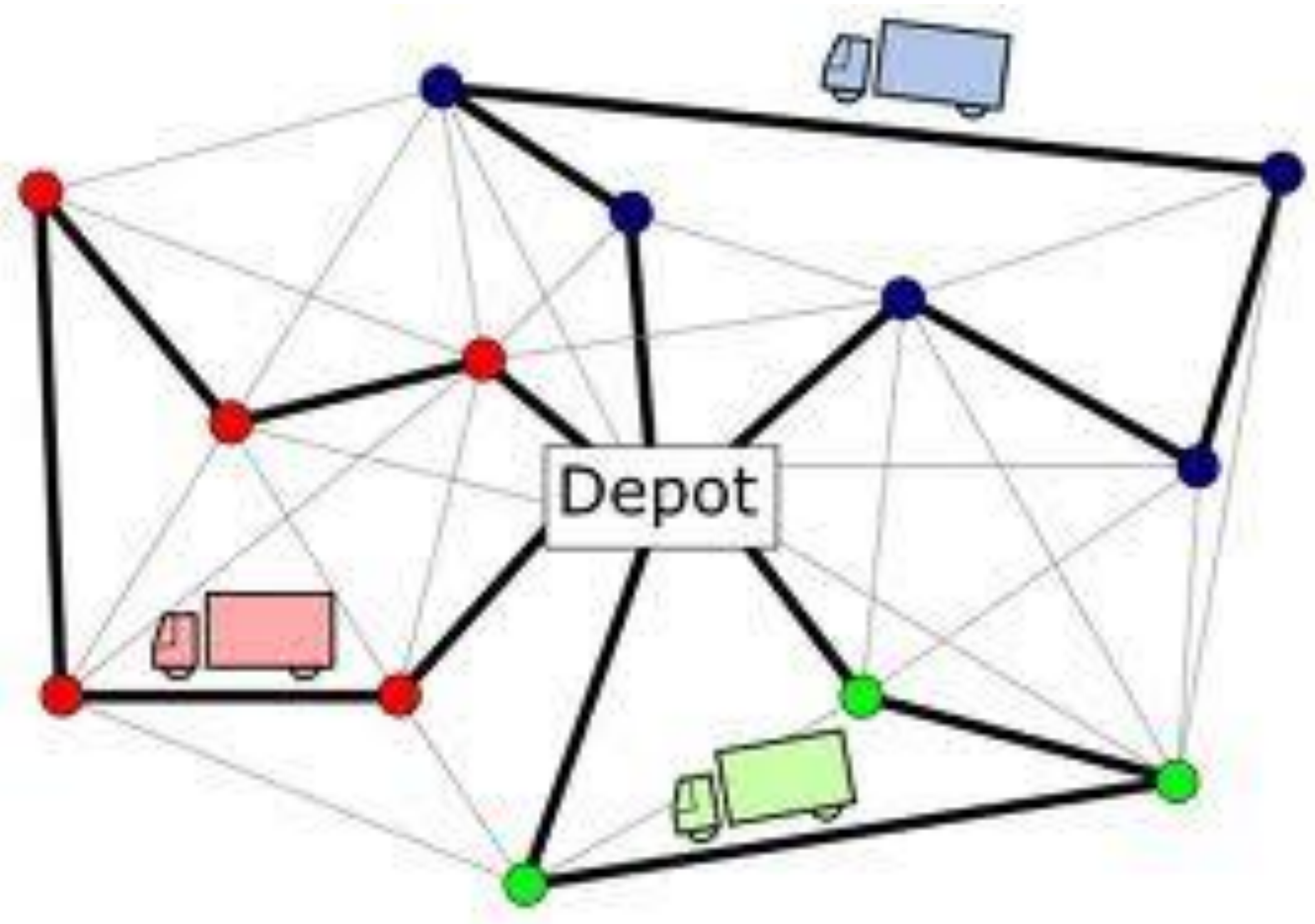


Image Source: https://www.r-bloggers.com/any-r-packages-to-solve-vehicle-routing-problem/

Pengembangan Model Matematis Permasalahan VRP

VRP with Time Windows

$$min \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} x_{ij}^{v} C_{ij}$$

$$\sum_{j \in N} \sum_{v \in V} x_{ij}^{v} = 1, \qquad \forall i \in N$$

$$\sum_{i \in N} \sum_{v \in V} x_{ij}^{v} = 1, \qquad \forall j \in N$$

$$\sum_{i \in N} x_{ik}^{v} - \sum_{j \in} x_{kj}^{v} = 0, \forall k \in N \cup M,\ v \in V$$

$$g_j^v \geq g_i^v + \left(x_{ij}^v - 1\right) Z + G_j, \forall v \in V, i, j \in N$$

$$g_j^v \leq H, \forall v \in V, j \in N$$

$$a_j^v \geq a_i^v + \left(x_{ij}^v - 1\right) Z + C_{ij}, \forall v \in V, i, j \in N$$

$$A_i \geq a_i^v \geq B_i, \forall v \in V, j \in N$$

$$x_{ij}^v \in \{1,0\}, \forall\ i, j \in N, v \in V$$

$$g_j^v \in R^+$$

Variable:
$\boldsymbol{x_{ij}^v}$: binary variable to indicate the path that goes from city i to city j by vehicle v is taken.
$\boldsymbol{g_j^v}$: accumulated vehicle load v at city j

Parameter:
$\boldsymbol{C_{ij}}$: cost of transporting from city i to city j.
$\boldsymbol{G_j}$: load of node j

Additional Variable:
$\boldsymbol{a_j^v}$: accumulated travel time of vehicle v at city j

Additional Parameter:
$\boldsymbol{A_i}$: earliest time to serve city i.
$\boldsymbol{B_i}$: latest time to serve city i

Tambahan Variable dan Pembatas

$$a_j^v \geq a_i^v + (x_{ij}^v - 1)Z + C_{ij}, \forall v \in V, i, j \in N$$
$$A_i \geq a_i^v \geq B_i, \forall v \in V, j \in N$$

Additional Variable:
$\boldsymbol{a_j^v}$: accumulated travel time of vehicle v at city j

Additional Parameter:
$\boldsymbol{A_i}$: earliest time to serve city i.
$\boldsymbol{B_i}$: latest time to serve city i

- Variable a_j^v digunakan untuk merekam informasi akumulasi waktu tempuh pada setiap titik kunjungan $j, j \in N$
- Akumulasi waktu kunjungan dapat direpresentasikan sebagai waktu kedatangan pada titik pelanggan j
- Kedatangan pelanggan a_j^v dapat dibatasi oleh rentang time window $[A_i, B_i]$

Contoh VRPTW

- Penambahan Parameter atau Data Time Window

Node	A	B
0	0	1000
1	300	600
2	120	300
3	150	400
4	200	400
5	300	350
6	200	500
7	50	500
8	200	400
9	0	500
10	0	500

- Penulisan pembatas baru

$$a_j^v \geq a_i^v + (x_{ij}^v - 1)Z + C_{ij}, \forall v \in V, i, j \in N$$

$$A_i \geq a_i^v \geq B_i, \forall v \in V, j \in N$$

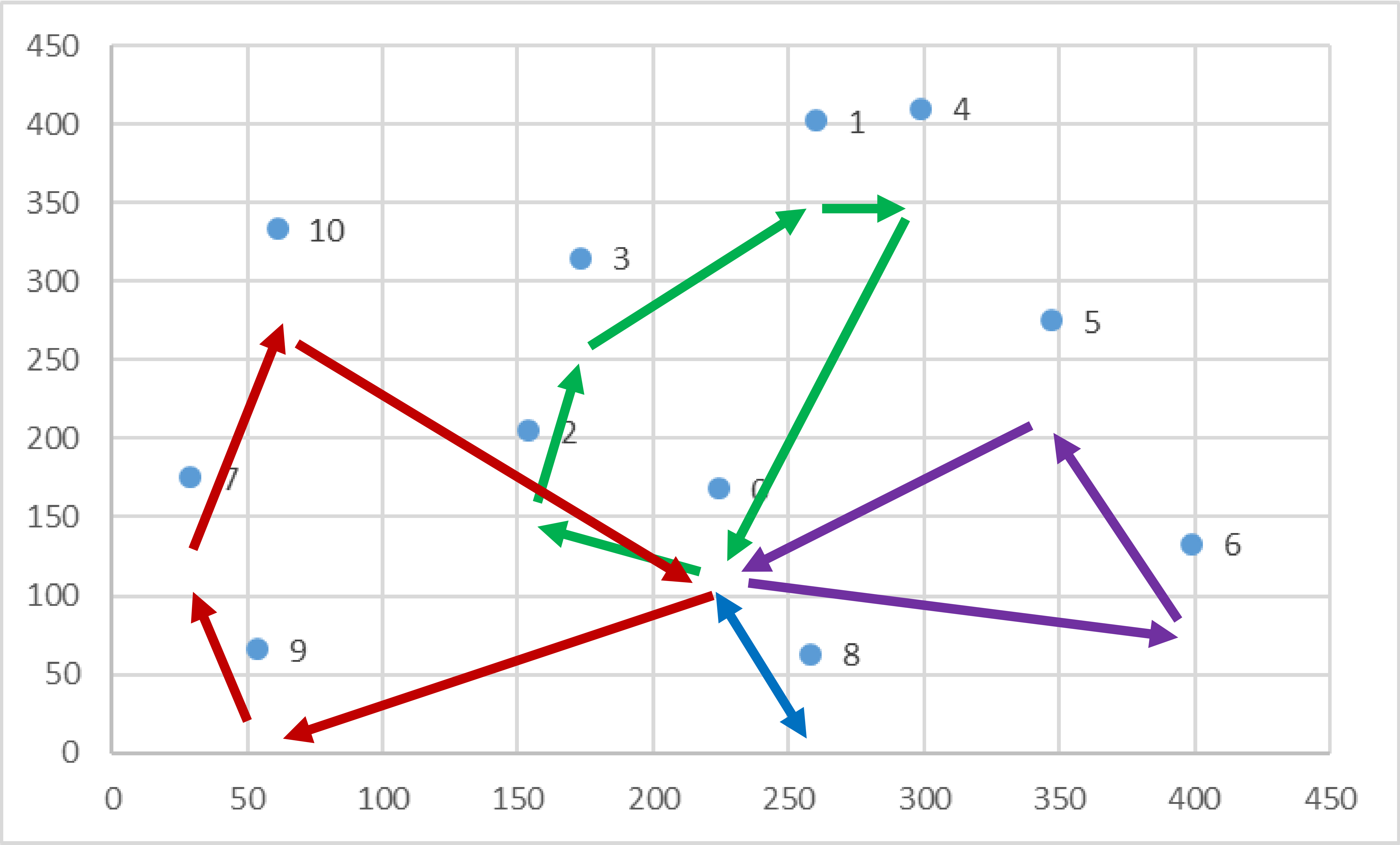
```
for i in Nd:
    for j in N:
        for v in V:
            m.addConstr(
                a[j,v] >= a[i,v] + T[i,j] + (x[i,j,v] - 1) * Z
                , "7[%s,%s,%s]"%(i,j,v))

for i in Nd:
    for v in V:
        m.addConstr(
            a[i,v] <= B[i]
            , "8[%s,%s]"%(i,v))


for i in Nd:
    for v in V:
        m.addConstr(
            a[i,v] >= A[i]
            , "9[%s,%s]"%(i,v))
```

Hasil Solusi menggunakan Gurobi Solver

```
     0     0 1764.00000    0   22 2356.00000 1764.00000  25.1%     -    0s
     0     0 1764.00000    0   16 2356.00000 1764.00000  25.1%     -    0s
H    0     0                    2199.0000000 1764.00000  19.8%     -    0s
     0     0 1764.00000    0   16 2199.00000 1764.00000  19.8%     -    0s
     0     0 1764.00000    0   16 2199.00000 1764.00000  19.8%     -    0s
     0     0 1764.00000    0   16 2199.00000 1764.00000  19.8%     -    0s
     0     2 1764.00000    0   16 2199.00000 1764.00000  19.8%     -    0s
*   68    21              10    2041.0000000 1852.78214  9.22%  17.2    0s
H   72    14                    2022.0000000 1852.78214  8.37%  16.6    0s

Cutting planes:
  Gomory: 12
  Cover: 2
  Implied bound: 15
  Clique: 21
  MIR: 26
  Zero half: 1

Explored 123 nodes (2262 simplex iterations) in 0.21 seconds
Thread count was 8 (of 8 available processors)

Solution count 4: 2022 2041 2199 2356

Optimal solution found (tolerance 1.00e-04)
Best objective 2.022000000000e+03, best bound 2.022000000000e+03, gap 0.0000%
```

```
[ V1 ] D => C8 => D
(  D ,  C8 ) : g(D) = 000.00 : a(D) = 000.00
(  C8 ,  D ) : g(C8) = 089.00 : a(C8) = 200.00
[ V2 ] D => C2 => C3 => C1 => C4 => D
(  D ,  C2 ) : g(D) = 000.00 : a(D) = 000.00
(  C1 ,  C4 ) : g(C1) = 433.00 : a(C1) = 360.00
(  C2 ,  C3 ) : g(C2) = 030.00 : a(C2) = 120.00
(  C3 ,  C1 ) : g(C3) = 394.00 : a(C3) = 237.00
(  C4 ,  D ) : g(C4) = 500.00 : a(C4) = 400.00
[ V3 ] D => C9 => C7 => C10 => D
(  D ,  C9 ) : g(D) = 000.00 : a(D) = 000.00
(  C7 , C10 ) : g(C7) = 166.00 : a(C7) = 310.00
(  C9 ,  C7 ) : g(C9) = 077.00 : a(C9) = 198.00
( C10 ,  D ) : g(C10) = 500.00 : a(C10) = 500.00
[ V4 ] D => C5 => C6 => D
(  D ,  C5 ) : g(D) = 000.00 : a(D) = 000.00
(  C5 ,  C6 ) : g(C5) = 083.00 : a(C5) = 300.00
(  C6 ,  D ) : g(C6) = 500.00 : a(C6) = 500.00
-------------------------------------------------
OPTIMAL VALUE= 2022.0
RUNTIME= 0.213428497314
```

Representasi Solusi




```
[ V1 ] D => C8 => D
(   D ,  C8 ) : g(D) = 000.00 : a(D) = 000.00
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(  C1 ,  C4 ) : g(C1) = 433.00 : a(C1) = 360.00
(  C2 ,  C3 ) : g(C2) = 030.00 : a(C2) = 120.00
(  C3 ,  C1 ) : g(C3) = 394.00 : a(C3) = 237.00
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(  C6 ,   D ) : g(C6) = 500.00 : a(C6) = 500.00
-------------------------------------------------
OPTIMAL VALUE= 2022.0
RUNTIME= 0.213428497314
```

Penambahan Time Window akan secara drastis merubah solusi dan menambah biaya