High School Mathematics Review

Mathematical Logic – First Term 2023-2024

MZI

School of Computing
Telkom University

SoC Tel-U

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Acknowledgements

This slide is compiled using the materials in the following sources:

1. *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
2. *Discrete Mathematics with Applications* (Chapter 4), 5th Edition, 2018, by S. S. Epp.
3. *GRE Math Review by ETS*.
4. Discrete Mathematics 1 (2012) slides at Fasilkom UI by B. H. Widjaja.
5. Discrete Mathematics 1 (2010) slides at Fasilkom UI by A. A. Krisnadhi.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to `<pleasedontspam>@telkomuniversity.ac.id`.

Contents

Contents

Integers

The set of integers, denoted by $\mathbb{Z}$, is a collection of the numbers: 1, 2, 3, and so on, together with their negatives (i.e., -1, -2, -3, and so on), and 0.

Divisibility

Let a and b be two integers with $a \neq 0$:

1. a divides b if there exists $k \in \mathbb{Z}$ such that $a \cdot k = b$,
2. if a divides b, then b is divisible by a, or equivalently, b is a multiple of a.

The condition a divides b can be written as $a|b$.

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1. 6 divides 12 because there exists $k = 2$ such that $6 \cdot 2 = 12$,
2. -3 divides 12 because there exists $k = -4$ such that $-3 \cdot (-4) = 12$,
3. 6 does not divide 9 because there exists no $k \in \mathbb{Z}$ such that $6 \cdot k = 9$ (the only value of k satisfying this condition is $k = 3/2 \notin \mathbb{Z}$),

Example

We have

1. 6 divides 12 because there exists $k = 2$ such that $6 \cdot 2 = 12$,
2. -3 divides 12 because there exists $k = -4$ such that $-3 \cdot (-4) = 12$,
3. 6 does not divide 9 because there exists no $k \in \mathbb{Z}$ such that $6 \cdot k = 9$ (the only value of k satisfying this condition is $k = 3/2 \notin \mathbb{Z}$),
4. -9 is divisible by 3 (or a multiple of 3) because 3 divides -9 (because $3 \cdot (-3) = -9$).

Exercise: Divisibility

Exercise

Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
5. 4 divides 0

Solution:

Exercise: Divisibility

Exercise

Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
5. 4 divides 0

Solution:

1. True, because there is $k = 9$ such that $6 \cdot 9 = 54$.

Exercise: Divisibility

Exercise

Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
5. 4 divides 0

Solution:

1. True, because there is $k = 9$ such that $6 \cdot 9 = 54$.
2. True, because there is $k = -9$ such that $3 \cdot (-9) = -27$.

Exercise: Divisibility

Exercise

Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
5. 4 divides 0

Solution:

1. True, because there is $k = 9$ such that $6 \cdot 9 = 54$.
2. True, because there is $k = -9$ such that $3 \cdot (-9) = -27$.
3. False, because there is no $k \in \mathbb{Z}$ such that $3 \cdot k = 91$ (since $91/3 = 30\frac{1}{3} \notin \mathbb{Z}$).

Exercise: Divisibility

Exercise

Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
5. 4 divides 0

Solution:

1. True, because there is $k = 9$ such that $6 \cdot 9 = 54$.
2. True, because there is $k = -9$ such that $3 \cdot (-9) = -27$.
3. False, because there is no $k \in \mathbb{Z}$ such that $3 \cdot k = 91$ (since $91/3 = 30\frac{1}{3} \notin \mathbb{Z}$).
4. False, because there is no $k \in \mathbb{Z}$ such that $-3 \cdot k = 17$ (since $17/-3 = -5\frac{2}{3} \notin \mathbb{Z}$).

Exercise: Divisibility

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Determine the truth of the following statements:

1. 6 divides 54
2. -27 is a multiple of 3
3. 3 divides 91
4. 17 is a multiple of -3
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Solution:

1. True, because there is $k = 9$ such that $6 \cdot 9 = 54$.
2. True, because there is $k = -9$ such that $3 \cdot (-9) = -27$.
3. False, because there is no $k \in \mathbb{Z}$ such that $3 \cdot k = 91$ (since $91/3 = 30\frac{1}{3} \notin \mathbb{Z}$).
4. False, because there is no $k \in \mathbb{Z}$ such that $-3 \cdot k = 17$ (since $17/-3 = -5\frac{2}{3} \notin \mathbb{Z}$).
5. True, because there is $k = 0$ such that $4 \cdot 0 = 0$.

Greatest Common Divisor, $\gcd$

The largest integer that divides both of two integers (not both are zero) is called the greatest common divisor of these integers.

Definition

Let $a, b \in \mathbb{Z}$, not both are zero. The largest integer d such that $d|a$ and $d|b$ is called the *greatest common divisor* of a and b. If d is the greatest common divisor of a and b, we denote d as $\gcd(a, b)$.

We infer that d is $\gcd(a, b)$ if d satisfies following condition:

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We infer that d is $\gcd(a, b)$ if d satisfies following condition:

1. $d|a$ and $d|b$,
2. if there is $c \in \mathbb{Z}$ such that $c|a$ and $c|b$, then $c|d$.

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
5. -3 and 0

Solution: Observe that

Exercise

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Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
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5. -3 and 0

Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
5. -3 and 0

Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) =$

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
5. -3 and 0

Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
5. -3 and 0

Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are

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1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are

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1. 24 and 36
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) =$

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Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
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Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) = 1$.
3. We have $120 =$

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1. 24 and 36
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) = 1$.
3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 =$

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) = 1$.
3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) =$

Exercise

Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
3. 120 and 500
4. -3 and -9
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) = 1$.
3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.

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1. 24 and 36
2. 17 and 22
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are

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Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
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1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are ± 1 and ± 3, the divisors of -9 are

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1. 24 and 36
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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are ± 1 and ± 3, the divisors of -9 are ± 1, ± 3, and ± 9, as a result $\gcd(-3, -9) =$

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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are ± 1 and ± 3, the divisors of -9 are ± 1, ± 3, and ± 9, as a result $\gcd(-3, -9) = 3$.

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1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are ± 1 and ± 3, the divisors of -9 are ± 1, ± 3, and ± 9, as a result $\gcd(-3, -9) = 3$.
5. The divisors of -3 are ± 1 and ± 3, since 0 is divisible by 3, then $\gcd(-3, 0) =$

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Determine the $\gcd$ of

1. 24 and 36
2. 17 and 22
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Solution: Observe that

1. The positive divisors of 24 are $1, 2, 3, 4, 6, 12, 24$, and the positive divisors of 36 are $1, 2, 3, 4, 6, 12, 18, 36$. Therefore $\gcd(24, 36) = 12$.
2. The positive divisors of 17 are 1 and 17, and the positive divisors of 22 are $1, 2, 11, 22$. Therefore $\gcd(17, 22) = 1$.
3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, therefore $\gcd(120, 500) = 2^{\min(3,2)} \cdot 3^{\min(1,0)} \cdot 5^{\min(1,3)} = 2^2 \cdot 5^1 = 20$.
4. The divisors of -3 are ± 1 and ± 3, the divisors of -9 are ± 1, ± 3, and ± 9, as a result $\gcd(-3, -9) = 3$.
5. The divisors of -3 are ± 1 and ± 3, since 0 is divisible by 3, then $\gcd(-3, 0) = 3$.

Least Common Multiple, lcm

The smallest integer which are the multiple of two positive integers is called the least common multiple of these numbers.

Definition

Let $a, b \in \mathbb{Z}^+$. The smallest integer c that is divisible by both a and b is called the *least common multiple* of a and b. If c is the least common multiple of a and b, then we denote c as $\mathrm{lcm}\,(a, b)$.

We infer that c is $\mathrm{lcm}\,(a, b)$ if c satisfies following condition:

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We infer that c is $\operatorname{lcm}(a, b)$ if c satisfies following condition:

1. $a|c$ and $b|c$,
2. if there is $d \in \mathbb{Z}$ such that $a|d$ and $b|d$, then $c|d$.

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
3. 120 and 500,

Solution: Observe that

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
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Solution: Observe that

1. The multiples of 24 are

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
3. 120 and 500,

Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
3. 120 and 500,

Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\mathrm{lcm}\,(24, 36) =$

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
3. 120 and 500,

Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\mathrm{lcm}\,(24, 36) = 72$.

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
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Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\operatorname{lcm}(24, 36) = 72$.
2. The multiples of 7 are

Exercise

Determine the lcm of

1. 24 and 36,
2. 7 and 3,
3. 120 and 500,

Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\mathrm{lcm}\,(24, 36) = 72$.
2. The multiples of 7 are $7, 14, 21, \ldots$, and the multiples of 3 are

Exercise

Determine the lcm of

1. 24 and 36,
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Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\text{lcm}\,(24, 36) = 72$.
2. The multiples of 7 are $7, 14, 21, \ldots$, and the multiples of 3 are $3, 9, 12, 15, 18, 21, \ldots$, therefore $\text{lcm}\,(7, 3) =$

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Determine the lcm of

1. 24 and 36,
2. 7 and 3,
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Solution: Observe that

1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\text{lcm}\,(24, 36) = 72$.
2. The multiples of 7 are $7, 14, 21, \ldots$, and the multiples of 3 are $3, 9, 12, 15, 18, 21, \ldots$, therefore $\text{lcm}\,(7, 3) = 21$.

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Determine the lcm of

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2. The multiples of 7 are $7, 14, 21, \ldots$, and the multiples of 3 are $3, 9, 12, 15, 18, 21, \ldots$, therefore $\mathrm{lcm}\,(7, 3) = 21$.
3. We have $120 =$

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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 =$

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3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, hence $\operatorname{lcm}(120, 500) =$

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1. The multiples of 24 are $24, 48, 72, 96, \ldots$, and the multiples of 36 are $36, 72, 108, \ldots$, therefore $\operatorname{lcm}(24, 36) = 72$.
2. The multiples of 7 are $7, 14, 21, \ldots$, and the multiples of 3 are $3, 9, 12, 15, 18, 21, \ldots$, therefore $\operatorname{lcm}(7, 3) = 21$.
3. We have $120 = 2^3 \cdot 3 \cdot 5$ and $500 = 2^2 \cdot 5^3$, hence $\operatorname{lcm}(120, 500) = 2^{\max(3,2)} \cdot 3^{\max(1,0)} \cdot 5^{\max(1,3)} = 2^3 \cdot 3 \cdot 5^3 = 3000$.

Even and Odd Numbers

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An integer n is even if there exists an integer k such that $n = 2k$; an integer n is odd if there exists and integer k such that $n = 2k + 1$.

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Observe that n is an even integer iff n is divisible by 2.

Example

The numbers -2, -4, 0, and 2020 are even.

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Example

The numbers -2, -4, 0, and 2020 are even. We have $-2 = 2\,(-1)$, $-4 = 2\,(-2)$, $0 = 2\,(0)$, and $2020 = 2\,(1010)$. The numbers -3, -7, 1, and 2021 are odd. We have $-3 =$

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The numbers -2, -4, 0, and 2020 are even. We have $-2 = 2(-1)$, $-4 = 2(-2)$, $0 = 2(0)$, and $2020 = 2(1010)$. The numbers -3, -7, 1, and 2021 are odd. We have $-3 = 2(-2) + 1$, $-7 = 2(-4) + 1$, $1 = (2)(0) + 1$, and $2021 = 2(1010) + 1$.

Exercise

Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 =$

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Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 = 2(-9) + 1$, so -17 is odd; $71 =$

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Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 = 2(-9) + 1$, so -17 is odd; $71 = 2(35) + 1$, so 71 is odd; $84 =$

Exercise

Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 = 2(-9) + 1$, so -17 is odd; $71 = 2(35) + 1$, so 71 is odd; $84 = 2(42)$, so 84 is even; $-1990 =$

Exercise

Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 = 2(-9) + 1$, so -17 is odd; $71 = 2(35) + 1$, so 71 is odd; $84 = 2(42)$, so 84 is even; $-1990 = 2(-995)$, so -1990 is even; and $-109 =$

Exercise

Determine whether each of these integers are even or odd: -17, 71, 84, -1990, and -109. Express in the form $2k$ or $2k+1$ for an integer k.

Solution: We have $-17 = 2(-9) + 1$, so -17 is odd; $71 = 2(35) + 1$, so 71 is odd; $84 = 2(42)$, so 84 is even; $-1990 = 2(-995)$, so -1990 is even; and $-109 = 2(-55) + 1$, so -109 is odd.

Perfect Square

Definition

An integer n is called a perfect square if there exists an integer b such that $n = b^2$.

Example

The numbers 4, 9, and 49 are perfect squares, because $4 =$

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The numbers 4, 9, and 49 are perfect squares, because $4 = 2^2$, $9 = 3^2$, and
49

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The numbers 4, 9, and 49 are perfect squares, because $4 = 2^2$, $9 = 3^2$, and $49 = 7^2$.

Perfect Square

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An integer n is called a perfect square if there exists an integer b such that $n = b^2$.

Example

The numbers 4, 9, and 49 are perfect squares, because $4 = 2^2$, $9 = 3^2$, and $49 = 7^2$. The numbers 7, 8, and 11 are not perfect squares, because there are no integers a, b, and c such that $7 = a^2$, $8 = b^2$, and $11 = c^2$.

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

1. $81 = 9^2$, so 81 is a perfect square;

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

1. $81 = 9^2$, so 81 is a perfect square;
2. $225 = 15^2$, so 225 is a perfect square;

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

1. $81 = 9^2$, so 81 is a perfect square;
2. $225 = 15^2$, so 225 is a perfect square;
3. -64 is not a perfect square (because $n^2 \geq 0$ for any integer n);

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

1. $81 = 9^2$, so 81 is a perfect square;
2. $225 = 15^2$, so 225 is a perfect square;
3. -64 is not a perfect square (because $n^2 \geq 0$ for any integer n);
4. 90 is not a perfect square (because $9^2 < 90 < 10^2$ and there is no integer between 9 and 10);

Exercise

Verify whether the following integers are perfect square or not: 81, 225, -64, 90, 0.

Solution: We have:

1. $81 = 9^2$, so 81 is a perfect square;
2. $225 = 15^2$, so 225 is a perfect square;
3. -64 is not a perfect square (because $n^2 \geq 0$ for any integer n);
4. 90 is not a perfect square (because $9^2 < 90 < 10^2$ and there is no integer between 9 and 10);
5. $0 = 0^2$, so 0 is a perfect square.

Contents

Fractions or Rational Numbers

Fractions/ Rational Numbers

A fraction or a rational number is a number of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

In a fraction $\frac{a}{b}$

1. a is called the **numerator**,
2. b is called the **denominator**.

A fraction $\frac{a}{b}$ is in **simplest form** if $\gcd(a, b) = 1$.

Example

We have:

1. $\frac{8}{18}$ is a rational number, the simplest form of $\frac{8}{18}$ is $\frac{4}{9}$, we have $\gcd(4,9)=1$.

Example

We have:

1. $\frac{8}{18}$ is a rational number, the simplest form of $\frac{8}{18}$ is $\frac{4}{9}$, we have $\gcd(4, 9) = 1$.
2. 25 is a rational number and it can be written as $\frac{25}{1}$, we have $\gcd(25, 1) = 1$.

Example

We have:

1. $\frac{8}{18}$ is a rational number, the simplest form of $\frac{8}{18}$ is $\frac{4}{9}$, we have $\gcd(4, 9) = 1$.
2. 25 is a rational number and it can be written as $\frac{25}{1}$, we have $\gcd(25, 1) = 1$.
3. $-\frac{5}{25}$ is a rational number, the simplest form of $-\frac{5}{25}$ is $-\frac{1}{5}$, we have $\gcd(-1, 5) = 1$.

Example

We have:

1. $\frac{8}{18}$ is a rational number, the simplest form of $\frac{8}{18}$ is $\frac{4}{9}$, we have $\gcd(4,9)=1$.
2. 25 is a rational number and it can be written as $\frac{25}{1}$, we have $\gcd(25,1)=1$.
3. $-\frac{5}{25}$ is a rational number, the simplest form of $-\frac{5}{25}$ is $-\frac{1}{5}$, we have $\gcd(-1,5)=1$.
4. -101 is a rational number and it can be written as $-\frac{101}{1}$, we have $\gcd(-101,1)=1$.

Exercise: Junior High Review

Exercise

Express each of these operation in a simplest rational number:

1. $\frac{1}{3} - \frac{2}{5}$
2. $\frac{21}{8} \times \frac{4}{9}$
3. $\frac{17}{8} : \frac{3}{4}$
4. $\frac{a}{b} + \frac{a^2}{b^2}$, where $b \neq 0$
5. $\left(\frac{a}{b} - \frac{b}{a}\right) : \left(\frac{1}{a} - \frac{1}{b}\right)$, where $a, b \neq 0$ and $a \neq b$

Solution:

1. $\frac{1}{3} - \frac{2}{5} =$

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Solution:

1. $\frac{1}{3} - \frac{2}{5} = -\frac{1}{15}$
2. $\frac{21}{8} \times \frac{4}{9} =$

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Solution:

1. $\frac{1}{3} - \frac{2}{5} = -\frac{1}{15}$
2. $\frac{21}{8} \times \frac{4}{9} = \frac{7}{6}$
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Contents

Powers (Exponents)

Powers (Exponents)

Powers (exponents) are used to denote the repeated multiplication of a number by itself, for instances:

1. $3^4 = (3)(3)(3)(3) = 81$,
2. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$.

In general, for $a \in \mathbb{R}$ and positive integer n

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n \text{ terms}}.$$

We also define

1. if $a \neq 0$, then $a^0 = 1$,
2. if $a \neq 0$ and n is a positive integer, then $a^{-n} = \frac{1}{a^n}$.

Roots

Roots

A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

1. $\sqrt{9} =$

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A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

1. $\sqrt{9} = 3$ because $3^2 = 9$,
2. $\sqrt{25} =$

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A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

1. $\sqrt{9} = 3$ because $3^2 = 9$,
2. $\sqrt{25} = 5$ because $5^2 = 25$.

The n-th root of a number a, denoted by $\sqrt[n]{a}$, is defined as a number r such that $r^n = a$. For instances:

1. $\sqrt[3]{8} =$

Roots

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A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

1. $\sqrt{9} = 3$ because $3^2 = 9$,
2. $\sqrt{25} = 5$ because $5^2 = 25$.

The n-th root of a number a, denoted by $\sqrt[n]{a}$, is defined as a number r such that $r^n = a$. For instances:

1. $\sqrt[3]{8} = 2$ because $2^3 = 8$,
2. $\sqrt[4]{81} =$

Roots

Roots

A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

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The n-th root of a number a, denoted by $\sqrt[n]{a}$, is defined as a number r such that $r^n = a$. For instances:

1. $\sqrt[3]{8} = 2$ because $2^3 = 8$,
2. $\sqrt[4]{81} = 3$ because $3^4 = 81$,
3. $\sqrt[3]{-27} =$

Roots

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A square root of a nonnegative number a, denoted by $\sqrt{a}$, is a (nonnegative) number r such that $r^2 = a$. For instances:

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The n-th root of a number a, denoted by $\sqrt[n]{a}$, is defined as a number r such that $r^n = a$. For instances:

1. $\sqrt[3]{8} = 2$ because $2^3 = 8$,
2. $\sqrt[4]{81} = 3$ because $3^4 = 81$,
3. $\sqrt[3]{-27} = -3$ because $(-3)^3 = -27$.

The n-th root of a number a or $\sqrt[n]{a}$ can also be written as $a^{\frac{1}{n}}$.

Properties of Exponents and Roots

Rules Regarding Square Roots Operations

Suppose $a > 0$ and $b > 0$.

No.	Rules	Examples
1	$(\sqrt{a})^2 = a$	

Properties of Exponents and Roots

Rules Regarding Square Roots Operations

Suppose $a > 0$ and $b > 0$.

No.	Rules	Examples
1	$(\sqrt{a})^2 = a$	$\left(\sqrt{3}\right)^2 = 3$
2	$\sqrt{a^2} = a$	

Properties of Exponents and Roots

Rules Regarding Square Roots Operations

Suppose $a > 0$ and $b > 0$.

No.	Rules	Examples
1	$\left(\sqrt{a}\right)^2 = a$	$\left(\sqrt{3}\right)^2 = 3$
2	$\sqrt{a^2} = a$	$\sqrt{2^2} = 2$
3	$\sqrt{a}\sqrt{b} = \sqrt{ab}$	

Properties of Exponents and Roots

Rules Regarding Square Roots Operations

Suppose $a > 0$ and $b > 0$.

No.	Rules	Examples
1	$\left(\sqrt{a}\right)^2 = a$	$\left(\sqrt{3}\right)^2 = 3$
2	$\sqrt{a^2} = a$	$\sqrt{2^2} = 2$
3	$\sqrt{a}\sqrt{b} = \sqrt{ab}$	$\sqrt{2}\sqrt{8} = \sqrt{16} = 4$
4	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	

Properties of Exponents and Roots

Rules Regarding Square Roots Operations

Suppose $a > 0$ and $b > 0$.

No.	Rules	Examples
1	$\left(\sqrt{a}\right)^2 = a$	$\left(\sqrt{3}\right)^2 = 3$
2	$\sqrt{a^2} = a$	$\sqrt{2^2} = 2$
3	$\sqrt{a}\sqrt{b} = \sqrt{ab}$	$\sqrt{2}\sqrt{8} = \sqrt{16} = 4$
4	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3.$

Rules Regarding Powers Operations

Suppose $x \neq 0$ and $y \neq 0$, a and b are integers.

No.	Rules	Examples
1	$x^{-a} = \frac{1}{x^a}$	

Rules Regarding Powers Operations

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Exercise: Properties of exponents and roots

Exercise

Simplify following expressions:

1. $\left(n^5\right)\left(n^{-3}\right)$
2. $\left(s^7\right)\left(t^7\right)$
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Solution:

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1. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
2. $(3a-2b)^2 = 9a^2 - 12ab + 4b^2$.