Predicate Logic 3: Translation From Natural Language to Predicate Formulas – Inference Rules for Quantified Formulas

Mathematical Logic – First Term 2023-2024

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Acknowledgements

This slide is compiled using the materials in the following sources:

1. *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
2. *Discrete Mathematics with Applications* (Chapter 3), 5th Edition, 2018, by S. S. Epp.
3. *Logic in Computer Science: Modelling and Reasoning about Systems* (Chapter 2), 2nd Edition, 2004, by M. Huth and M. Ryan.
4. *Mathematical Logic for Computer Science* (Chapter 5, 6), 2nd Edition, 2000, by M. Ben-Ari.
5. Discrete Mathematics 1 (2012) slides in Fasilkom UI by B. H. Widjaja.
6. Mathematical Logic slides in Telkom University by A. Rakhmatsyah and B. Purnama.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to `<pleasedontspam>@telkomuniversity.ac.id`.

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Contents

Translating Human Language to Predicate Logic

The process of translating a particular human language into predicate logic is conducted using following steps:

1. Defining appropriate domain(s) for the natural language sentence in predicate logic.
2. Defining appropriate predicate(s) for the translation.
3. Expressing the sentence using the previously defined predicate(s).

Translation Example 1

Suppose we want to express the following sentence in a predicate formula: "every student in Mathematical Logic class also learns Calculus".
First answer:

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Suppose we want to express the following sentence in a predicate formula: "every student in Mathematical Logic class also learns Calculus".
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1. Define $D := \{x \mid x \text{ is a student in Mathematical Logic class}\}$.

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1. Define $D := \{x \mid x \text{ is a student in Mathematical Logic class}\}$.
2. Define $P(x) := x$ learns Calculus.

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Suppose we want to express the following sentence in a predicate formula: "every student in Mathematical Logic class also learns Calculus".
First answer:

1. Define $D := \{x \mid x \text{ is a student in Mathematical Logic class}\}$.
2. Define $P(x) := x$ learns Calculus.
3. Therefore the sentence "every student in Mathematical Logic class also learns Calculus" can be expressed as:

Translation Example 1

Suppose we want to express the following sentence in a predicate formula: "every student in Mathematical Logic class also learns Calculus".
First answer:

1. Define $D := \{x \mid x \text{ is a student in Mathematical Logic class}\}$.
2. Define $P(x) := x$ learns Calculus.
3. Therefore the sentence "every student in Mathematical Logic class also learns Calculus" can be expressed as: $\boxed{\forall x P(x)}$.

Second answer:

Second answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.

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2. Define $Q(x) := x$ is a student in Mathematical Logic class.

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1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $Q(x) := x$ is a student in Mathematical Logic class.
3. Define $R(x) := x$ learns Calculus.

Second answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $Q(x) := x$ is a student in Mathematical Logic class.
3. Define $R(x) := x$ learns Calculus.
4. Therefore the sentence "every student in Mathematical Logic class also learns Calculus" can be expressed as:

Second answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $Q(x) := x$ is a student in Mathematical Logic class.
3. Define $R(x) := x$ learns Calculus.
4. Therefore the sentence "every student in Mathematical Logic class also learns Calculus" can be expressed as: $\boxed{\forall x\,(Q(x) \rightarrow R(x))}$.

Remark

Second answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $Q(x) := x$ is a student in Mathematical Logic class.
3. Define $R(x) := x$ learns Calculus.
4. Therefore the sentence "every student in Mathematical Logic class also learns Calculus" can be expressed as: $\boxed{\forall x\,(Q(x) \rightarrow R(x))}$.

Remark

The sentence "every student in Mathematical Logic class also learns Calculus" cannot be expressed as $\forall x\,(Q(x) \wedge R(x))$ because this formula means "every student in informatics major is a student in Mathematical Logic class and also learns Calculus".

Translation Example 2

Suppose we want to express following sentence in a predicate formula: “there is an informatics student who loves mathematics”.
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Suppose we want to express following sentence in a predicate formula: "there is an informatics student who loves mathematics".

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1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.

Translation Example 2

Suppose we want to express following sentence in a predicate formula: "there is an informatics student who loves mathematics".
First answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $P(x) := x$ loves mathematics.

Translation Example 2

Suppose we want to express following sentence in a predicate formula: "there is an informatics student who loves mathematics".
First answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $P(x) := x$ loves mathematics.
3. Therefore the sentence "there is an informatics student who loves mathematics" can be expressed as:

Translation Example 2

Suppose we want to express following sentence in a predicate formula: "there is an informatics student who loves mathematics".
First answer:

1. Define $D := \{x \mid x \text{ is a student in informatics major}\}$.
2. Define $P(x) := x$ loves mathematics.
3. Therefore the sentence "there is an informatics student who loves mathematics" can be expressed as: $\boxed{\exists x P(x)}$.

Second answer:

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1. Define $D := \{x \mid x \text{ is a student}\}$

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2. Define $Q(x) := x$ is an informatics student
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Second answer:

1. Define $D := \{x \mid x \text{ is a student}\}$
2. Define $Q(x) := x$ is an informatics student
3. Define $R(x) := x$ loves mathematics
4. Therefore the sentence "there is an informatics student who loves mathematics" can be expressed as: $\boxed{\exists x\,(Q(x) \wedge R(x))}$.

Remark

The sentence "there is an informatics student who loves mathematics" cannot be expressed as $\exists x\,(Q(x) \rightarrow R(x))$ because the formula $\exists x\,(Q(x) \rightarrow R(x))$ **is also true** if there is a student who loves mathematics, although this student is not an informatics major.

Exercise: Translation to Predicate Formulas (1)

Exercise

Express the sentence: "if someone is a male who has a child, then he is a father", using the domain $D := \{x \mid x \text{ is a human}\}$ and following predicates:

1. $\mathrm{Male}(x) := x$ is a male, $\mathrm{Child}(x) := x$ has a child, and $\mathrm{Father}(x) := x$ is a father
2. $\mathrm{Male}(x) := x$ is a male, $\mathrm{Parent}(x, y) := x$ is a parent of y, and $\mathrm{Father}(x) := x$ is a father

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2. $\text{Male}\,(x) := x$ is a male, $\text{Parent}\,(x, y) := x$ is a parent of y, and $\text{Father}\,(x) := x$ is a father

Solution:

1. The sentence can be rewritten as: "for any person x, if x is a male and x has a child, then x is a father". Using the predicates in no. 1 we can express this sentence in predicate formula as:

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Solution:

1. The sentence can be rewritten as: "for any person x, if x is a male and x has a child, then x is a father". Using the predicates in no. 1 we can express this sentence in predicate formula as: $\forall x\,(\text{Male}(x) \wedge \text{Child}(x) \rightarrow \text{Father}(x))$.

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2. The sentence can be rewritten as: "for any person x, if x is a male and there is a y such that x is a parent of y, then x is a father". Using the predicates in no. 2 we can express this sentence in predicate formula as:

Exercise: Translation to Predicate Formulas (1)

Exercise

Express the sentence: "if someone is a male who has a child, then he is a father", using the domain $D := \{x \mid x \text{ is a human}\}$ and following predicates:

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2. The sentence can be rewritten as: "for any person x, if x is a male and there is a y such that x is a parent of y, then x is a father". Using the predicates in no. 2 we can express this sentence in predicate formula as: $\forall x\,(\text{Male}(x) \wedge \exists y \text{Parent}(x, y) \rightarrow \text{Father}(x))$.

Exercise: Translation to Predicate Formulas (2)

Exercise

Express each of the following sentences in a predicate formula:

1. Bob is the best friend of Alice.
2. Every person has a best friend.
3. Alice has only one best friend.
4. Every person has only one best friend.

You may only use the domain $D := \{x \mid x \text{ is a human}\}$, predicate $\mathrm{Friend}\,(x, y)$ which means "x has a best friend whose name is y", predicate $=$ ('equal to'), and predicate $\neq$ ('not equal to').

Solution:

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Solution:

1. The first sentence "Bob is the best friend of Alice" is equivalent to "Alice has a best friend whose name is Bob". In predicate logic this sentence can be expressed as:

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Express each of the following sentences in a predicate formula:

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1. The first sentence "Bob is the best friend of Alice" is equivalent to "Alice has a best friend whose name is Bob". In predicate logic this sentence can be expressed as: $\mathrm{Friend}\,(Alice, Bob)$.

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Solution:

1. The first sentence "Bob is the best friend of Alice" is equivalent to "Alice has a best friend whose name is Bob". In predicate logic this sentence can be expressed as: $\text{Friend}(Alice, Bob)$.
2. The sentence 2 can be rewritten as: "for every person x, there is a person y, such that $\text{Friend}(x, y)$". Therefore, the sentence can be expressed in predicate logic as:

Exercise: Translation to Predicate Formulas (2)

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Express each of the following sentences in a predicate formula:

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Solution:

1. The first sentence "Bob is the best friend of Alice" is equivalent to "Alice has a best friend whose name is Bob". In predicate logic this sentence can be expressed as: $\text{Friend}(Alice, Bob)$.
2. The sentence 2 can be rewritten as: "for every person x, there is a person y, such that $\text{Friend}(x, y)$". Therefore, the sentence can be expressed in predicate logic as: $\forall x\,(\exists y \text{Friend}(x, y))$ or $\forall x \exists y \text{Friend}(x, y)$.

3. The sentence 3 can be rewritten as: “there is a person x who is a best friend of Alice, and every person y who is not equal to x is not Alice’s best friend”. This sentence can be expressed in predicate logic as:

3. The sentence 3 can be rewritten as: "there is a person x who is a best friend of Alice, and every person y who is not equal to x is not Alice's best friend". This sentence can be expressed in predicate logic as:

$$\exists x\, (\mathrm{Friend}\,(Alice, x) \wedge \forall y\, ((x \neq y) \rightarrow \neg \mathrm{Friend}\,(Alice, y))) \text{ , or}$$

$$\exists x \forall y\, (\mathrm{Friend}\,(Alice, x) \wedge ((x \neq y) \rightarrow \neg \mathrm{Friend}\,(Alice, y)))$$

3. The sentence 3 can be rewritten as: "there is a person x who is a best friend of Alice, and every person y who is not equal to x is not Alice's best friend". This sentence can be expressed in predicate logic as:

$$\exists x\left(\text{Friend}\left(Alice,x\right)\wedge\forall y\left(\left(x\neq y\right)\rightarrow\neg\text{Friend}\left(Alice,y\right)\right)\right),\text{ or}$$
$$\exists x\forall y\left(\text{Friend}\left(Alice,x\right)\wedge\left(\left(x\neq y\right)\rightarrow\neg\text{Friend}\left(Alice,y\right)\right)\right)$$

Sentence 3 can also be rewritten as: "there is a person x who is a best friend of Alice, and for every person y who claims that he/she is Alice's best friend, then x and y is the same person. This sentence can be expressed in predicate logic as:

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$$\exists x\,(\mathrm{Friend}\,(Alice, x) \wedge \forall y\,((x \neq y) \to \neg\mathrm{Friend}\,(Alice, y)))\text{ , or}$$
$$\exists x \forall y\,(\mathrm{Friend}\,(Alice, x) \wedge ((x \neq y) \to \neg\mathrm{Friend}\,(Alice, y)))$$

Sentence 3 can also be rewritten as: "there is a person x who is a best friend of Alice, and for every person y who claims that he/she is Alice's best friend, then x and y is the same person. This sentence can be expressed in predicate logic as:

$$\exists x\,(\mathrm{Friend}\,(Alice, x) \wedge \forall y\,(\mathrm{Friend}\,(Alice, y) \to (x = y)))\text{ , or}$$
$$\exists x \forall y\,(\mathrm{Friend}\,(Alice, x) \wedge (\mathrm{Friend}\,(Alice, y) \to (x = y)))\,.$$

4. The sentence 4 can be rewritten as: "for every person x, there exists a person y, such that $\text{Friend}(x, y)$, and every person z which is not equal to y is not a best friend of x".
This sentence can be expressed in predicate logic as:

4. The sentence 4 can be rewritten as: "for every person x, there exists a person y, such that $\text{Friend}(x, y)$, and every person z which is not equal to y is not a best friend of x".
   This sentence can be expressed in predicate logic as:

$$\forall x \exists y \left(\text{Friend}(x, y) \wedge \forall z \left((z \neq y) \rightarrow \neg\text{Friend}(x, z)\right)\right), \text{ or}$$

$$\forall x \exists y \forall z \left(\text{Friend}(x, y) \wedge \left((z \neq y) \rightarrow \neg\text{Friend}(x, z)\right)\right).$$

4. The sentence 4 can be rewritten as: "for every person x, there exists a person y, such that $\text{Friend}(x, y)$, and every person z which is not equal to y is not a best friend of x".
This sentence can be expressed in predicate logic as:

$$\forall x \exists y \left(\text{Friend}(x, y) \wedge \forall z \left((z \neq y) \rightarrow \neg \text{Friend}(x, z)\right)\right), \text{ or}$$
$$\forall x \exists y \forall z \left(\text{Friend}(x, y) \wedge \left((z \neq y) \rightarrow \neg \text{Friend}(x, z)\right)\right).$$

Sentence 4 can also be rewritten as: "for every person x there exists a person y such that $\text{Friend}(x, y)$, and every person z, if we have $\text{Friend}(x, z)$, then it must be $y = z$. This sentence can be expressed in predicate logic as:

4. The sentence 4 can be rewritten as: "for every person x, there exists a person y, such that $\mathrm{Friend}(x, y)$, and every person z which is not equal to y is not a best friend of x".
This sentence can be expressed in predicate logic as:

$$\forall x \exists y \left(\mathrm{Friend}(x, y) \wedge \forall z \left((z \neq y) \rightarrow \neg \mathrm{Friend}(x, z)\right)\right), \text{ or}$$
$$\forall x \exists y \forall z \left(\mathrm{Friend}(x, y) \wedge \left((z \neq y) \rightarrow \neg \mathrm{Friend}(x, z)\right)\right).$$

Sentence 4 can also be rewritten as: "for every person x there exists a person y such that $\mathrm{Friend}(x, y)$, and every person z, if we have $\mathrm{Friend}(x, z)$, then it must be $y = z$. This sentence can be expressed in predicate logic as:

$$\forall x \left(\exists y \left(\mathrm{Friend}(x, y) \wedge \forall z \left(\mathrm{Friend}(x, z) \rightarrow y = z\right)\right)\right), \text{ or}$$
$$\forall x \exists y \left(\mathrm{Friend}(x, y) \wedge \forall z \left(\mathrm{Friend}(x, z) \rightarrow y = z\right)\right), \text{ or}$$
$$\forall x \exists y \forall z \left(\mathrm{Friend}(x, y) \wedge \left(\mathrm{Friend}(x, z) \rightarrow y = z\right)\right).$$

Contents

Exercise

Let $\text{Likes}(x, y)$ be a predicate defined over the domain $D_1 \times D_2 = \{(x, y) \mid x \in D_1, y \in D_2\}$ with $D_1 = \{x \mid x$ is a student$\}$ and $D_2 = \{y \mid y$ is a food$\}$. Suppose $Ammy, Ben, Carl \in D_1$ and $burger, crepes, pie, pizza \in D_2$. Translate each of the following sentences into a correct predicate formula.

1. Ammy and Ben like burger.
2. Carl likes crepes or pie.
3. Everyone likes burger.
4. Carl likes every food.
5. Someone likes pie.
6. Someone likes every food.
7. There is a food which is liked by everyone.
8. Everyone likes at least one food.
9. Everyone who likes burger also likes pizza.
10. There is a food which is liked by Ammy and Ben.

Solution:

1. Ammy and Ben like burger.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger).$
2. Carl likes crepes or pie.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
3. Everyone likes burger.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
3. Everyone likes burger. $\forall x \text{Likes}\,(x, burger)$.
4. Carl likes every food.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
3. Everyone likes burger. $\forall x \text{Likes}\,(x, burger)$.
4. Carl likes every food. $\forall y \text{Likes}\,(Carl, y)$.
5. Someone likes pie.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
3. Everyone likes burger. $\forall x \text{Likes}\,(x, burger)$.
4. Carl likes every food. $\forall y \text{Likes}\,(Carl, y)$.
5. Someone likes pie. $\exists x \text{Likes}\,(x, pie)$.
6. Someone likes every food.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
3. Everyone likes burger. $\forall x \text{Likes}\,(x, burger)$.
4. Carl likes every food. $\forall y \text{Likes}\,(Carl, y)$.
5. Someone likes pie. $\exists x \text{Likes}\,(x, pie)$.
6. Someone likes every food. $\exists x \forall y \text{Likes}\,(x, y)$.
7. There is a food which is liked by everyone.

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1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
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7. There is a food which is liked by everyone. $\exists y \forall x \text{Likes}\,(x, y)$.
8. Everyone likes at least one food.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
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7. There is a food which is liked by everyone. $\exists y \forall x \text{Likes}\,(x, y)$.
8. Everyone likes at least one food. $\forall x \exists y \text{Likes}\,(x, y)$.
9. Everyone who likes burger also likes pizza.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
2. Carl likes crepes or pie. $\text{Likes}\,(Carl, crepes) \vee \text{Likes}\,(Carl, pie)$.
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8. Everyone likes at least one food. $\forall x \exists y \text{Likes}\,(x, y)$.
9. Everyone who likes burger also likes pizza.
   $\forall x\,(\text{Likes}\,(x, burger) \rightarrow \text{Likes}\,(x, pizza))$.
10. There is a food which is liked by Ammy and Ben.

Solution:

1. Ammy and Ben like burger. $\text{Likes}\,(Ammy, burger) \wedge \text{Likes}\,(Ben, burger)$.
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   $\forall x\,(\text{Likes}\,(x, burger) \rightarrow \text{Likes}\,(x, pizza))$.
10. There is a food which is liked by Ammy and Ben.
    $\exists y\,(\text{Likes}\,(Ammy, y) \wedge \text{Likes}\,(Ben, y))$.

Contents

Negation of a Quantified Sentence

Exercise

The correct negation of the sentence: "there is an informatics student who doesn't use computer everyday" is:

a. There is an informatics student who uses computer everyday.
b. Every informatics student uses computer everyday.
c. Every informatics student uses computer at least one day.
d. Every informatics student doesn't use computer everyday.
e. There is no correct answer among a, b, c, and d.

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The correct negation of the sentence: "there is an informatics student who doesn't use computer everyday" is:

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Solution to Exercise

1. We first define $D_1 := \{x \mid x \text{ is a student}\}$, $D_2 := \{y \mid y \text{ is a day}\}$.

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3. We define $\texttt{Computer}\,(x, y) :=$"$x$ uses computer in day y", $\texttt{Computer}$ is a binary predicate over domain $D_1 \times D_2$.

The sentence: "there is an informatics who doesn't use computer everyday" can be expressed in predicate formula as:

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The sentence: "there is an informatics who doesn't use computer everyday" can be expressed in predicate formula as:

$$\exists x\,(\texttt{Informatics}\,(x) \wedge \neg\forall y\,(\texttt{Computer}\,(x, y)))$$

The negation of the previous formula is

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$\equiv$

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[De Morgan's law for $\forall$]

$$\equiv$$

The negation of the previous formula is

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\equiv\; & \forall x\,(\neg\texttt{Informatics}\,(x) \vee \neg\neg\forall y\,(\texttt{Computer}\,(x,y))) \\
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\equiv\; & \forall x \left(\neg\texttt{Informatics}\left(x\right) \lor \neg\neg\forall y \left(\texttt{Computer}\left(x,y\right)\right)\right) \\
& \text{[De Morgan's law for } \land \text{]} \\
\equiv\; & \forall x \left(\neg\texttt{Informatics}\left(x\right) \lor \forall y \left(\texttt{Computer}\left(x,y\right)\right)\right) \quad \text{[double negation law]} \\
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Contents

Rules of Inference for Quantified Formulas

Since first-order predicate logic is an extension of propositional logic, all rules of inference in propositional logic are also applicable in first-order predicate logic. In addition, rules of inference in predicate logic are also equipped with rules of inference for quantified formulas, which comprise:

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Since first-order predicate logic is an extension of propositional logic, all rules of inference in propositional logic are also applicable in first-order predicate logic. In addition, rules of inference in predicate logic are also equipped with rules of inference for quantified formulas, which comprise:

- universal instantiation
- universal generalization
- existential instantiation
- existential generalization
- universal modus ponens
- universal modus tollens

Universal Instantiation

Universal Instantiation

Suppose P is a unary predicate defined over a domain D and $c \in D$, then

$$\frac{\forall x \ P(x)}{\therefore P(c)}$$

Observe that $\forall x P(x) \rightarrow P(c)$ is a valid formula, hence we have $\forall x P(x) \Rightarrow P(c)$.

Example

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Example

Suppose Albert is an informatics major,

Every informatics major takes Mathematical Logic

∴ Albert takes Mathematical Logic

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- This rule is used implicitly in many proofs in mathematics.
- However, the error of adding unwarranted assumptions about c when universal generalization is used is not uncommon in incorrect reasoning.

Example of Universal Generalization

Suppose domain for x is the set of all non-zero integers and $P(x)$ is the statement "$x^2 \geq 1$". We want to prove that $\forall x\ P(x)$ is true by proving that $P(c)$ is true for arbitrary non-zero integer c.

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Therefore, for every non-zero integer c we have "$c^2 \geq 1$". In other words $\forall x\ \left(x^2 \geq 1\right)$.

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$$\frac{\exists x\ P(x)}{\therefore P(c), \text{ for some (particular) } c \in D}$$

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- In this case, c is not arbitrary, but rather it must be a particular c which makes $P(c)$ is **true**.
- In some mathematical facts, we have no knowledge of what c is, only that it exists.

Example of Existential Instantiation

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In this case the value of c is 0.

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Also observe that

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In this case the value of c is 3.
Furthermore, it is easy to prove that there is no other value than 0 and 3 which makes $P(c)$ is true.

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Suppose P is a unary predicate defined over a domain D and c is a particular element in D, then

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- This rules states that, if there is a particular element c in the domain D which makes $P(c)$ is **true**, then $\exists x \; P(x)$ is **true** as well.

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Exercise: Inferences in Predicate Logic (1)

Exercise

Suppose we have following premises: "every student in Mathematical Logic class also takes Calculus", "Andre is a student in Mathematical Logic class", and "Benny doesn't take Calculus".

Verify whether from these premises we can conclude the statement: "Andre takes Calculus and Benny is not the student in Mathematical Logic class".

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3. We shall verify whether the premises infer the conclusion $\texttt{Calc}\,(\text{Andre}) \wedge \neg\texttt{MathLog}\,(\text{Benny})$.

Inference steps:

1. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Calc}\,(x))$ (premise)
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4. $\texttt{MathLog}\,(\text{Andre}) \to \texttt{Calc}\,(\text{Andre})$ (universal instantiation of 1)

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4. $\texttt{MathLog}\,(\text{Andre}) \rightarrow \texttt{Calc}\,(\text{Andre})$ (universal instantiation of 1)
5. $\texttt{Calc}\,(\text{Andre})$ (modus ponens of 4 and 2)

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5. $\texttt{Calc}\,(\text{Andre})$ (modus ponens of 4 and 2)
6. $\texttt{MathLog}\,(\text{Benny}) \rightarrow \texttt{Calc}\,(\text{Benny})$ (universal instantiation of 1)
7. $\neg\texttt{MathLog}\,(\text{Benny})$ (modus tollens of 6 and 3)

3. We shall verify whether the premises infer the conclusion $\texttt{Calc}\,(\text{Andre}) \wedge \neg\texttt{MathLog}\,(\text{Benny})$.

Inference steps:

1. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Calc}\,(x))$ (premise)
2. $\texttt{MathLog}\,(\text{Andre})$ (premise)
3. $\neg\texttt{Calc}\,(\text{Benny})$ (premise)
4. $\texttt{MathLog}\,(\text{Andre}) \rightarrow \texttt{Calc}\,(\text{Andre})$ (universal instantiation of 1)
5. $\texttt{Calc}\,(\text{Andre})$ (modus ponens of 4 and 2)
6. $\texttt{MathLog}\,(\text{Benny}) \rightarrow \texttt{Calc}\,(\text{Benny})$ (universal instantiation of 1)
7. $\neg\texttt{MathLog}\,(\text{Benny})$ (modus tollens of 6 and 3)
8. $\texttt{Calc}\,(\text{Andre}) \wedge \neg\texttt{MathLog}\,(\text{Benny})$ (conjunction of 5 and 7)

Universal Modus Ponens and Modus Tollens

Suppose P and Q are two unary predicates which are evaluated in domain D and $a \in D$.

Universal Modus Ponens

$$\forall x \;\; (P(x) \rightarrow Q(x))$$
$$P(a), \text{ for an } a \in D$$

$$\therefore$$

Universal Modus Ponens and Modus Tollens

Suppose P and Q are two unary predicates which are evaluated in domain D and $a \in D$.

Universal Modus Ponens

$$\forall x \ \ (P(x) \rightarrow Q(x))$$
$$P(a), \text{ for an } a \in D$$
$$\therefore Q(a)$$

Universal Modus Tollens

$$\forall x \ \ (P(x) \rightarrow Q(x))$$
$$\neg Q(a), \text{ for an } a \in D$$
$$\therefore$$

Universal Modus Ponens and Modus Tollens

Suppose P and Q are two unary predicates which are evaluated in domain D and $a \in D$.

Universal Modus Ponens

$$\forall x \; (P(x) \rightarrow Q(x))$$
$$P(a), \text{ for an } a \in D$$
$$\overline{}$$
$$\therefore Q(a)$$

Universal Modus Tollens

$$\forall x \; (P(x) \rightarrow Q(x))$$
$$\neg Q(a), \text{ for an } a \in D$$
$$\overline{}$$
$$\therefore \neg P(a)$$

Universal modus ponens is a derived rule which is obtained from modus ponens and universal instantiation. Analogous derivation also applies to universal modus ponens.

Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $P(a)$ for an $a \in D$.

Universal Modus Ponens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $P(a)$ (premise)

Universal modus ponens is a derived rule which is obtained from modus ponens and universal instantiation. Analogous derivation also applies to universal modus ponens.

Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $P(a)$ for an $a \in D$.

Universal Modus Ponens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $P(a)$ (premise)
3. $P(a) \rightarrow Q(a)$ (universal instantiation of 1)

Universal modus ponens is a derived rule which is obtained from modus ponens and universal instantiation. Analogous derivation also applies to universal modus ponens.

Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $P(a)$ for an $a \in D$.

Universal Modus Ponens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $P(a)$ (premise)
3. $P(a) \rightarrow Q(a)$ (universal instantiation of 1)
4. $Q(a)$ (modus ponens of 3 and 1).

Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $\neg Q(b)$ for a $b \in D$.

Universal Modus Tollens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $\neg Q(b)$ (premise)

Universal modus ponens is a derived rule which is obtained from modus ponens and universal instantiation. Analogous derivation also applies to universal modus ponens.
Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $P(a)$ for an $a \in D$.

Universal Modus Ponens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $P(a)$ (premise)
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1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
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Universal modus ponens is a derived rule which is obtained from modus ponens and universal instantiation. Analogous derivation also applies to universal modus ponens.

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Universal Modus Ponens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $P(a)$ (premise)
3. $P(a) \rightarrow Q(a)$ (universal instantiation of 1)
4. $Q(a)$ (modus ponens of 3 and 1).

Suppose we have the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $\neg Q(b)$ for a $b \in D$.

Universal Modus Tollens

1. $\forall x\,(P(x) \rightarrow Q(x))$ (premise)
2. $\neg Q(b)$ (premise)
3. $P(b) \rightarrow Q(b)$ (universal instantiation of 1)
4. $\neg P(b)$ (modus tollens of 3 and 1).

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$"$n > 4$", and $Q(n) :=$"$n^2 < 2^n$".

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$ "$n > 4$", and $Q(n) :=$ "$n^2 < 2^n$".
2. Our assumption can be expressed as

Exercise: Inferences in Predicate Logic (2)

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Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

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Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$ "$n > 4$", and $Q(n) :=$ "$n^2 < 2^n$".
2. Our assumption can be expressed as $\forall n \; (P(n) \rightarrow Q(n))$.

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$ "$n > 4$", and $Q(n) :=$ "$n^2 < 2^n$".
2. Our assumption can be expressed as $\forall n \; (P(n) \rightarrow Q(n))$.
3. Observe that $P(100)$ is **true** because $100 > 4$.

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$ "$n > 4$", and $Q(n) :=$ "$n^2 < 2^n$".
2. Our assumption can be expressed as $\forall n \ (P(n) \rightarrow Q(n))$.
3. Observe that $P(100)$ is **true** because $100 > 4$.
4. Using universal modus ponens, we infer that $Q(100)$ is **true** or

Exercise: Inferences in Predicate Logic (2)

Exercise

Using the assumption that following statement is true:

"for every positive number n, if $n > 4$, then $n^2 < 2^n$",

prove that $100^2 < 2^{100}$.

Solution:

1. Suppose the universe of discourse is the set of all positive integers, $P(n) :=$ "$n > 4$", and $Q(n) :=$ "$n^2 < 2^n$".
2. Our assumption can be expressed as $\forall n \ (P(n) \to Q(n))$.
3. Observe that $P(100)$ is **true** because $100 > 4$.
4. Using universal modus ponens, we infer that $Q(100)$ is **true** or $100^2 < 2^{100}$.

Exercise: Inferences in Predicate Logic (3)

Exercise

Suppose we have following premises: "a student in Mathematical Logic class has not read the textbook" and "every student in Mathematical Logic class passed the midterm"

Verify whether these premises infer the conclusion "someone who passed the midterm has not read the textbook"

Solution:

Exercise: Inferences in Predicate Logic (3)

Exercise

Suppose we have following premises: "a student in Mathematical Logic class has not read the textbook" and "every student in Mathematical Logic class passed the midterm"

Verify whether these premises infer the conclusion "someone who passed the midterm has not read the textbook"

Solution:

1. Suppose the universe of discourse is $D := \{x \mid x \text{ is a student}\}$ and the predicates are: $\texttt{MathLog}\,(x)$:= "x is a student in Mathematical Logic class", $\texttt{TextBook}\,(x)$:= "x has read the textbook", $\texttt{Passed}\,(x)$:= "x passed the midterm".

Exercise: Inferences in Predicate Logic (3)

Exercise

Suppose we have following premises: "a student in Mathematical Logic class has not read the textbook" and "every student in Mathematical Logic class passed the midterm"

Verify whether these premises infer the conclusion "someone who passed the midterm has not read the textbook"

Solution:

1. Suppose the universe of discourse is $D := \{x \mid x \text{ is a student}\}$ and the predicates are: $\texttt{MathLog}(x) :=$ "x is a student in Mathematical Logic class", $\texttt{TextBook}(x) :=$ "x has read the textbook", $\texttt{Passed}(x) :=$ "x passed the midterm".
2. The premises can be expressed in predicate formulas as:

Exercise: Inferences in Predicate Logic (3)

Exercise

Suppose we have following premises: "a student in Mathematical Logic class has not read the textbook" and "every student in Mathematical Logic class passed the midterm"
Verify whether these premises infer the conclusion "someone who passed the midterm has not read the textbook"

Solution:

1. Suppose the universe of discourse is $D := \{x \mid x \text{ is a student}\}$ and the predicates are: $\texttt{MathLog}\,(x) :=$ "x is a student in Mathematical Logic class", $\texttt{TextBook}\,(x) :=$ "x has read the textbook", $\texttt{Passed}\,(x) :=$ "x passed the midterm".
2. The premises can be expressed in predicate formulas as:

$$\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$$

Exercise: Inferences in Predicate Logic (3)

Exercise

Suppose we have following premises: "a student in Mathematical Logic class has not read the textbook" and "every student in Mathematical Logic class passed the midterm"

Verify whether these premises infer the conclusion "someone who passed the midterm has not read the textbook"

Solution:

1. Suppose the universe of discourse is $D := \{x \mid x \text{ is a student}\}$ and the predicates are: $\texttt{MathLog}(x) :=$ "x is a student in Mathematical Logic class", $\texttt{TextBook}(x) :=$ "x has read the textbook", $\texttt{Passed}(x) :=$ "x passed the midterm".
2. The premises can be expressed in predicate formulas as:

$$\exists x \left(\texttt{MathLog}(x) \wedge \neg\texttt{TextBook}(x)\right)$$
$$\forall x \left(\texttt{MathLog}(x) \rightarrow \texttt{Passed}(x)\right)$$

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)
5. $\texttt{MathLog}\,(\mathrm{c}) \rightarrow \texttt{Passed}\,(\mathrm{c})$ (for the same c as in no. 3 and 4, obtained from universal instantiation of 2)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)
5. $\texttt{MathLog}\,(\mathrm{c}) \rightarrow \texttt{Passed}\,(\mathrm{c})$ (for the same c as in no. 3 and 4, obtained from universal instantiation of 2)
6. $\texttt{Passed}\,(\mathrm{c})$ (modus ponens of 5 and 4)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \land \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \land \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \to \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \land \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)
5. $\texttt{MathLog}\,(\mathrm{c}) \to \texttt{Passed}\,(\mathrm{c})$ (for the same c as in no. 3 and 4, obtained from universal instantiation of 2)
6. $\texttt{Passed}\,(\mathrm{c})$ (modus ponens of 5 and 4)
7. $\neg\texttt{TextBook}\,(\mathrm{c})$ (simplification of 3)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)
5. $\texttt{MathLog}\,(\mathrm{c}) \rightarrow \texttt{Passed}\,(\mathrm{c})$ (for the same c as in no. 3 and 4, obtained from universal instantiation of 2)
6. $\texttt{Passed}\,(\mathrm{c})$ (modus ponens of 5 and 4)
7. $\neg\texttt{TextBook}\,(\mathrm{c})$ (simplification of 3)
8. $\texttt{Passed}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (conjunction of 6 and 7)

3. We shall verify whether the premises infer the conclusion $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$.

Inference steps:

1. $\exists x\,(\texttt{MathLog}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (premise)
2. $\forall x\,(\texttt{MathLog}\,(x) \rightarrow \texttt{Passed}\,(x))$ (premise)
3. $\texttt{MathLog}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (for particular $\mathrm{c} \in D$, obtained from existential instantiation from 1)
4. $\texttt{MathLog}\,(\mathrm{c})$ (simplification of 3)
5. $\texttt{MathLog}\,(\mathrm{c}) \rightarrow \texttt{Passed}\,(\mathrm{c})$ (for the same c as in no. 3 and 4, obtained from universal instantiation of 2)
6. $\texttt{Passed}\,(\mathrm{c})$ (modus ponens of 5 and 4)
7. $\neg\texttt{TextBook}\,(\mathrm{c})$ (simplification of 3)
8. $\texttt{Passed}\,(\mathrm{c}) \wedge \neg\texttt{TextBook}\,(\mathrm{c})$ (conjunction of 6 and 7)
9. $\exists x\,(\texttt{Passed}\,(x) \wedge \neg\texttt{TextBook}\,(x))$ (existential generalization from 8)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P\,(x) \rightarrow (Q\,(x) \wedge S\,(x)))$ and $\forall x\,(P\,(x) \wedge R\,(x))$ infer the conclusion $\forall x\,(R\,(x) \wedge S\,(x))$.

Solution:

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)
6. $Q(c) \wedge S(c)$ (modus ponens of 3 and 5)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)
6. $Q(c) \wedge S(c)$ (modus ponens of 3 and 5)
7. $S(c)$ (simplification of 6)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)
6. $Q(c) \wedge S(c)$ (modus ponens of 3 and 5)
7. $S(c)$ (simplification of 6)
8. $R(c)$ (simplification of 4)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)
6. $Q(c) \wedge S(c)$ (modus ponens of 3 and 5)
7. $S(c)$ (simplification of 6)
8. $R(c)$ (simplification of 4)
9. $R(c) \wedge S(c)$ (conjunction of 7 and 8)

Inference in Predicate Logic (4)

Exercise

Verify whether the premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ infer the conclusion $\forall x\,(R(x) \wedge S(x))$.

Solution:

1. $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ (premise)
2. $\forall x\,(P(x) \wedge R(x))$ (premise)
3. $P(c) \rightarrow (Q(c) \wedge S(c))$ (universal instantiation of 1 using instance c which is an arbitrary element in the domain)
4. $P(c) \wedge R(c)$ (universal instantiation of 2 using instance c which is an arbitrary element in the domain)
5. $P(c)$ (simplification of 4)
6. $Q(c) \wedge S(c)$ (modus ponens of 3 and 5)
7. $S(c)$ (simplification of 6)
8. $R(c)$ (simplification of 4)
9. $R(c) \wedge S(c)$ (conjunction of 7 and 8)
10. $\forall x\,(R(x) \wedge S(x))$ (universal generalization of 9, because c is an arbitrary element in the domain)

Exercise: Inference in Predicate Logic (5 – Supplementary)

Exercise

Suppose we have following premises: "every teaching assistant for Mathematical Logic course is a third or a fourth year student", "every fourth year student has passed Algorithm Analysis and Artificial Intelligence courses", "Raymond is a teaching assistant for Mathematical Logic course who has passed Algorithm Analysis, but he hasn't passed Artificial Intelligence course".
Verify whether these premises infer the conclusion "there is a teaching assistant for Mathematical Logic course who is a third year student".

Solution:

Exercise: Inference in Predicate Logic (5 – Supplementary)

Exercise

Suppose we have following premises: "every teaching assistant for Mathematical Logic course is a third or a fourth year student", "every fourth year student has passed Algorithm Analysis and Artificial Intelligence courses", "Raymond is a teaching assistant for Mathematical Logic course who has passed Algorithm Analysis, but he hasn't passed Artificial Intelligence course".
Verify whether these premises infer the conclusion "there is a teaching assistant for Mathematical Logic course who is a third year student".

Solution:

1. Suppose the universe of discourse is $D := \{x \mid x \text{ is a student}\}$ and the predicates are $\texttt{Assistant}(x) :=$ "x is a teaching assistant for Mathematical Logic course", $\texttt{Third}(x) :=$ "x is a third year student", $\texttt{Fourth}(x) :=$ "x is a fourth year student, $\texttt{AA}(x) :=$ "x has passed Algorithm Analysis course", and $\texttt{AI}(x) :=$ "x has passed Artificial Intelligence course".

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$$\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$$

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$$\forall x\,(\texttt{Assistant}\,(x) \to \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$$
$$\forall x\,(\texttt{Fourth}\,(x) \to \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$$

3. The premises can be expressed in predicate formulas as:

$$\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$$
$$\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$$
$$\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$$

4. We shall verity whether the premises infer the conclusion

3. The premises can be expressed in predicate formulas as:

$$\forall x \left(\mathtt{Assistant}\left(x\right) \rightarrow \mathtt{Third}\left(x\right) \vee \mathtt{Fourth}\left(x\right)\right)$$
$$\forall x \left(\mathtt{Fourth}\left(x\right) \rightarrow \mathtt{AA}\left(x\right) \wedge \mathtt{AI}\left(x\right)\right)$$
$$\mathtt{Assistant}\left(\text{Raymond}\right) \wedge \mathtt{AA}\left(\text{Raymond}\right) \wedge \neg\mathtt{AI}\left(\text{Raymond}\right)$$

4. We shall verity whether the premises infer the conclusion $\exists x \left(\mathtt{Assistant}\left(x\right) \wedge \mathtt{Third}\left(x\right)\right)$.

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)

1. $\forall x\,(\texttt{Assistant}\,(x) \to \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
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3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \to \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \to \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)

1. $\forall x\,(\texttt{Assistant}(x) \rightarrow \texttt{Third}(x) \vee \texttt{Fourth}(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}(x) \rightarrow \texttt{AA}(x) \wedge \texttt{AI}(x))$ (premise)
3. $\texttt{Assistant}(\text{Raymond}) \wedge \texttt{AA}(\text{Raymond}) \wedge \neg\texttt{AI}(\text{Raymond})$ (premise)
4. $\texttt{Assistant}(\text{Raymond}) \rightarrow \texttt{Third}(\text{Raymond}) \vee \texttt{Fourth}(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}(\text{Raymond}) \rightarrow \texttt{AA}(\text{Raymond}) \wedge \texttt{AI}(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}(\text{Raymond}) \vee \texttt{Fourth}(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}(\text{Raymond}) \wedge \neg\texttt{AI}(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}(\text{Raymond}) \vee \neg\texttt{AI}(\text{Raymond})$ (addition from 9)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}\,(\text{Raymond}) \vee \neg\texttt{AI}\,(\text{Raymond})$ (addition from 9)
11. $\neg\,(\texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond}))$ (De Morgan's law from 10)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}\,(\text{Raymond}) \vee \neg\texttt{AI}\,(\text{Raymond})$ (addition from 9)
11. $\neg\,(\texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond}))$ (De Morgan's law from 10)
12. $\neg\texttt{Fourth}\,(\text{Raymond})$ (modus tollens from 5 and 11)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
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3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}\,(\text{Raymond}) \vee \neg\texttt{AI}\,(\text{Raymond})$ (addition from 9)
11. $\neg\,(\texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond}))$ (De Morgan's law from 10)
12. $\neg\texttt{Fourth}\,(\text{Raymond})$ (modus tollens from 5 and 11)
13. $\texttt{Third}\,(\text{Raymond})$ (disjunctive syllogism from 7 and 12)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}\,(\text{Raymond}) \vee \neg\texttt{AI}\,(\text{Raymond})$ (addition from 9)
11. $\neg\,(\texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond}))$ (De Morgan's law from 10)
12. $\neg\texttt{Fourth}\,(\text{Raymond})$ (modus tollens from 5 and 11)
13. $\texttt{Third}\,(\text{Raymond})$ (disjunctive syllogism from 7 and 12)
14. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{Third}\,(\text{Raymond})$ (conjunction from 6 and 13)

1. $\forall x\,(\texttt{Assistant}\,(x) \rightarrow \texttt{Third}\,(x) \vee \texttt{Fourth}\,(x))$ (premise)
2. $\forall x\,(\texttt{Fourth}\,(x) \rightarrow \texttt{AA}\,(x) \wedge \texttt{AI}\,(x))$ (premise)
3. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (premise)
4. $\texttt{Assistant}\,(\text{Raymond}) \rightarrow \texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (universal instantiation from 1)
5. $\texttt{Fourth}\,(\text{Raymond}) \rightarrow \texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond})$ (universal instantiation from 2)
6. $\texttt{Assistant}\,(\text{Raymond})$ (simplification from 3)
7. $\texttt{Third}\,(\text{Raymond}) \vee \texttt{Fourth}\,(\text{Raymond})$ (modus ponens from 4 and 6)
8. $\texttt{AA}\,(\text{Raymond}) \wedge \neg\texttt{AI}\,(\text{Raymond})$ (simplification from 3)
9. $\neg\texttt{AI}\,(\text{Raymond})$ (simplification from 8)
10. $\neg\texttt{AA}\,(\text{Raymond}) \vee \neg\texttt{AI}\,(\text{Raymond})$ (addition from 9)
11. $\neg\,(\texttt{AA}\,(\text{Raymond}) \wedge \texttt{AI}\,(\text{Raymond}))$ (De Morgan's law from 10)
12. $\neg\texttt{Fourth}\,(\text{Raymond})$ (modus tollens from 5 and 11)
13. $\texttt{Third}\,(\text{Raymond})$ (disjunctive syllogism from 7 and 12)
14. $\texttt{Assistant}\,(\text{Raymond}) \wedge \texttt{Third}\,(\text{Raymond})$ (conjunction from 6 and 13)
15. $\exists x\,(\texttt{Assistant}\,(x) \wedge \texttt{Third}\,(x))$ (existential generalization from 14)