Pigeonhole Principle

Discrete Mathematics – Second Term 2022-2023

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Acknowledgements

This slide is composed based on the following materials:

1. *Discrete Mathematics and Its Applications*, 8th Edition, 2019, by K. H. Rosen (main).
2. *Discrete Mathematics with Applications* , 5th Edition, 2018, by S. S. Epp.
3. *Mathematics for Computer Science*. MIT, 2010, by E. Lehman, F. T. Leighton, A. R. Meyer.
4. Slide for Matematika Diskret 2 (2012). Fasilkom UI, by B. H. Widjaja.
5. Slide for Matematika Diskret. Telkom University, by B. Purnama.

Some of the pictures are taken from the above resources. This slide is intended for academic purpose at FIF Telkom University. If you have any suggestions/comments/questions related with the material on this slide, send an email to `<pleasedontspam>@telkomuniversity.ac.id`.

Contents

Contents

Problem of Taking Minimum Amounts

Notice the following problems.

Problem of Taking Minimum Amounts

1. In a dark room there is a container containing only three kinds of socks, namely: red, white, and black socks. What is the minimum number of socks should we take to ensure that we obtain a pair of socks of the same color? (We cannot bring any light source.)
2. Determine the minimum number of people required in a room to ensure that at least **three of them** are born at the same month.
3. PIN of ATM of a bank consists of six digits of number $(0-9)$. If the bank has 50 millions of customers, what is the minimum number of people must be gathered to ensure that at least two of them have identical PIN?

Pigeonhole Principle (PhP)

Picture taken from `Wikipedia`.

Pigeonhole Principle/ Dirichlet Box Principle (PhP/DBP)

If $(k+1)$ or more objects ("pigeons") are placed into k boxes ("holes"), then there is **at least one box** containing **two or more** of the objects.

Proof

Pigeonhole Principle/ Dirichlet Box Principle (PhP/DBP)

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Suppose the pigeonhole principle is not correct, then each box at most contains one object.

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Proof

Suppose the pigeonhole principle is not correct, then each box at most contains one object. Since there are only k boxes, then in general there will be at most k objects, which contradicts to the initial $k+1$ objects. □

Some Examples of PhP

Theorem (existence of injective total function)

Given two finite sets A and B, if $|A| > |B|$ then **it is impossible to have an injective total function** from A to B.

Theorem (existence of surjective total function)

Given two finite sets A and B, if $|A| < |B|$ then **it is impossible to have a surjective total function** from A to B.

Example

If there are 40 chairs ("holes") in a class and 42 students ("pigeons"), then there must be some students that do not get the chair.

Example

A school will have a dance party. If there are m girls and n boys in the class with $m < n$, then (maybe)

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A school will have a dance party. If there are m girls and n boys in the class with $m < n$, then (maybe)

1. there are some boys that will not come to the dance party because he has no pair,
2. every boys come to the dance party and at least one boy chooses a girl who does not come from the same school.

Identifying the "Pigeon" and "Its Hole"

Socks in the dark room

Container in the dark room with three kinds of socks, what is the least number of socks that should be taken in order to have a pair of matching socks?

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The important thing that we should consider when we try to solve the pigeonhole problems is about the “pigeon” and “its hole”. Sometimes we have to construct the “holes” that we need.

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1. If arbitrary five distinct numbers are taken from A, then there is a pair of numbers whose sum is equal to 9.
2. If arbitrary four distinct numbers are take from A, then there is a pair of numbers whose sum is equal to 9.

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For problem no. 1, suppose we take five numbers: $1, 3, 5, 7, 8$, then we have $1 + 8 = 9$. If the five numbers are: $2, 3, 5, 7, 8$, then we have $2 + 7 = 9$. Does this condition remains true for arbitrary five numbers taken from A?

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Generalized Pigeonhole Principle

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If N objects ("pigeon") are placed into k boxes ("holes"), then **at least one box** contains $\left\lceil \frac{N}{k} \right\rceil$ objects.

Proof of the Generalized Pigeonhole Principle

To prove the generalized pigeonhole principle, we will use the following lemma.

Lemma

For every $x \in \mathbb{R}$ we have $\lceil x \rceil < x + 1$ (or in other words $\lceil x \rceil - 1 < x$).

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$$k \cdot \left(\left\lceil \frac{N}{k} \right\rceil - 1 \right) < k \left(\frac{N}{k} \right) = N,$$

which means that the total objects is less than N. This is a contradiction with the given N objects. □

Some Examples of the Generalized Pigeonhole Principle

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   There are 12 months in a year, according to the generalized pigeonhole principle there are $\left\lceil \frac{25}{12} \right\rceil = 3$ people that are born at the same month.

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Telkom University use following grade indexes: $\mathbf{A}$, $\mathbf{AB}$, $\mathbf{B}$, $\mathbf{BC}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{T}$. What is the minimum number of students required in a course to ensure that at least twelve of them have the same index?

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Solution: Notice that there are eight possible grade indexes.

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Solution: Notice that there are eight possible grade indexes. To have at least twelve students with the same indexes, we have to find the minimum value of N such that $\left\lceil \frac{N}{8} \right\rceil = 12$. The minimum value of N can be obtained as follows

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$$N = (12-1) \cdot \underbrace{8}_{\text{8 kind of values}} + 1 = 89.$$

Notice that 89 is the minimum value of N that satisfies $\left\lceil \frac{N}{8} \right\rceil = 12$, because $\left\lceil \frac{89}{8} \right\rceil = \left\lceil \frac{88}{8} + \frac{1}{8} \right\rceil = \left\lceil 11\frac{1}{8} \right\rceil =$

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Contents

Problem 2

Exercise

1. Human blood can be classified into four types, namely: **A**, **B**, **AB**, and **O**. Each blood type has two kind of rhesus, namely $+$ or $-$. How many people that are needed to ensure that at least five of them have identical blood type and rhesus?
2. How many binary string of length 4 required to ensure that there are three identical strings?
3. The grade point average (GPA) is a three digits number of the form $a.bc$. The value $a.bc$ is not less than 0.00 and not more than 4.00. How many students required to ensure that five of them have identical GPA?

Solution: Problem 2 No. 1 & No. 2

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Using the product rule, the number of different binary strings of length 4 is $2^4 = 16$ strings. Hence, we need to find the minimum value of N so that $\left\lceil \frac{N}{16} \right\rceil =$

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