Predicate Logic 1: Motivation – Parse Tree

Mathematical Logic – First Term 2023-2024

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School of Computing
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SoC Tel-U

October 2023

Acknowledgements

This slide is compiled using the materials in the following sources:

1. *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
2. *Discrete Mathematics with Applications* (Chapter 3), 5th Edition, 2018, by S. S. Epp.
3. *Logic in Computer Science: Modelling and Reasoning about Systems* (Chapter 2), 2nd Edition, 2004, by M. Huth and M. Ryan.
4. *Mathematical Logic for Computer Science* (Chapter 5, 6), 2nd Edition, 2000, by M. Ben-Ari.
5. Discrete Mathematics 1 (2012) slides in Fasilkom UI by B. H. Widjaja.
6. Mathematical Logic slides in Telkom University by A. Rakhmatsyah and B. Purnama.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to `<pleasedontspam>@telkomuniversity.ac.id`.

Contents

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In the above examples, we don't see the relationship between p, q, and r, although all of these propositions state that "someone" is a student.

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Atomic Propositions in Predicate Logic

By using predicate logic, atomic propositions in our previous example have similar structures. Suppose we have

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All of these three propositions can be denoted respectively as $\texttt{Student}\,(Alex)$, $\texttt{Student}\,(Bernard)$, and $\texttt{Student}\,(Calvin)$. In these propositions, $\texttt{Student}$ is a predicate and $Alex$, $Bernard$, $Calvin$ are called constants. In these examples, $\texttt{Student}$ is a predicate with arity

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All of these three propositions can be denoted respectively as $\texttt{Student}\,(Alex)$, $\texttt{Student}\,(Bernard)$, and $\texttt{Student}\,(Calvin)$. In these propositions, Student is a predicate and $Alex$, $Bernard$, $Calvin$ are called constants. In these examples, Student is a predicate with arity 1 and the domain D can be a collection of all people in the world.

To express "x is a student" in predicate logic, we can write $\texttt{Student}\,(x)$.

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These propositions can be denoted respectively as $\texttt{Likes}\,(Alex, crepes)$, $\texttt{Likes}\,(Bernard, meatball)$, and $\texttt{Likes}\,(Calvin, pizza)$. In these propositions, , $\texttt{Likes}$ is a predicate with arity 2 and the domain of the predicate can be $D_1 \times D_2 = \{(x, y) \mid x \text{ is a person and } y \text{ is a food}\}$. This means D_1 is a collection of all people and D_2 is the collection of all foods. The order of the domain cannot be swapped over, $D_1 \times D_2$ is not equal to $D_2 \times D_1$.

To denote "(person) x likes (food) y", we write $\texttt{Likes}\,(x, y)$.

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"Alex is an informatics freshman"
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Remark

Predicate logic covered in this Mathematical Logic course is also called as *first-order predicate logic* or simply *first-order logic*. In this type of logic, quantification is applied to variables representing elements in particular domains (this will be discussed later in the slides).

Predicate as a function (1)

A predicate with arity n can be considered as a function from $D_1 \times D_2 \times \cdots \times D_n$ to $\{\mathrm{F}, \mathrm{T}\}$, where $D_1 \times D_2 \times \cdots \times D_n$ is a set of ordered tuple $(d_1, d_2, \ldots, d_n)$ with $d_i \in D_i$ for each $i = 1, 2, \ldots, n$.

Example

A unary predicate P with $P(x)$ denotes "$x > 2021$" can be considered as a function

$$P : D \to \{\mathrm{F}, \mathrm{T}\},$$

where D is the set of observed numbers. The truth values of $P(2021)$ and $P(2022)$ according to $P(x)$ are obtained as follows:

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Example

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$$Q : D \times D \to \{\mathrm{F}, \mathrm{T}\},$$

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Predicate as a function (3)

Example

A ternary predicate R with $R(x, y, z)$ denotes "$x + y = z$" can be considered as a function

$$R : D \times D \times D \to \{\mathrm{F}, \mathrm{T}\},$$

where D is the set of observed numbers. The truth values of $R(1, 2, 3)$ and $R(3, 2, 1)$ according to $R(x, y, z)$ are obtained as follows:

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- $R(3, 2, 1) \equiv (3 + 2 = 1) \equiv \mathrm{F}$ because $3 + 2 = 1$ is false.

We've seen the methods for verifying the truth values of predicates with arity 1, 2, and 3.

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- a proposition in propositional logic which we've learned earlier can be considered as a predicate with arity 0.

Contents

Quantification and Quantifier

In a predicate, there are two types of quantification which can be applied to variables:

1. *universal quantification*
2. *existential quantification*

These quantifications express the extent to which range a predicate is true over a range of elements. In English, the words *all*, *some*, *many*, *none*, and *few* are used in quantification.

Universal Quantification

Universal Quantification

Universal quantification for predicate $P(x)$ is the statement

"$P(x)$ for **all** (**every**) element x in the domain D"

This statement is denoted symbolically as

$\forall x \in D\ P(x)$, or
$\forall x\ P(x)$, if D is clear from context.

$P(x)$ is the *scope* of quantification $\forall x$.
The above formulation is usually read as

"For all (every) x in D we have $P(x)$", or
"$P(x)$ is true for every x in the universe of discourse"

The symbol $\forall$ is called the *universal quantifier*.

More About Universal Quantification

If the domain D is finite, for example, suppose $D = \{a_1, a_2, \ldots, a_n\}$, then we have

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If the domain D is finite, for example, suppose $D = \{a_1, a_2, \ldots, a_n\}$, then we have

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If the domain D is finite, for example, suppose $D = \{a_1, a_2, \dots, a_n\}$, then we have

$$\forall x \; P(x) \equiv P(a_1) \land P(a_2) \land \cdots \land P(a_n)$$

$\forall x \; P(x)$ is **false** if there is (at least) one x in D that makes $P(x)$ **false**.

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The value x which makes $\forall x \; P(x)$ false is called the *counterexample* of the statement $\forall x \; P(x)$.

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Suppose there are three people in a particular classroom, Alice, Bob, and Charlie.

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Suppose there are three people in a particular classroom, Alice, Bob, and Charlie. The statement "everyone in the classroom is a student" can be expressed as "Alice, Bob, and Charlie are students in the classroom".

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Suppose there are three people in a particular classroom, Alice, Bob, and Charlie. The statement "everyone in the classroom is a student" can be expressed as "Alice, Bob, and Charlie are students in the classroom". The statement "everyone in the classroom is a student" is false if **at least one** of Alice, Bob, or Charlie **is not** a student.

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Example

Suppose there are three people in a particular classroom, Alice, Bob, and Charlie. The statement "everyone in the classroom is a student" can be expressed as "Alice, Bob, and Charlie are students in the classroom". The statement "everyone in the classroom is a student" is false if **at least one** of Alice, Bob, or Charlie **is not** a student. Suppose, for example, Bob is not a student in the classroom, then Bob is the *counterexample* of the statement "everyone in the classroom is a student".

Existential Quantification

Existential Quantification

Existential quantification for predicate $P(x)$ is the statement

"$P(x)$ for **some (at least one)** element x in the domain D"

This statement is denoted symbolically as

$\exists x \in D \; P(x)$, or
$\exists x \; P(x)$, if D is clear from context.

$P(x)$ is the *scope* of quantification $\exists x$.
The above formulation is usually read as

"There is an x in D such that $P(x)$", or
"There is at least one x in the universe of discourse such that $P(x)$"

The symbol $\exists$ is called the *existential quantifier*.

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If the domain D is finite, for example, suppose $D = \{a_1, a_2, \ldots, a_n\}$, then we have

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If the domain D is finite, for example, suppose $D = \{a_1, a_2, \ldots, a_n\}$, then we have

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Example

Suppose there are three people in a particular classroom, Alice, Bob, and Charlie.

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If the domain D is finite, for example, suppose $D = \{a_1, a_2, \ldots, a_n\}$, then we have

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Suppose there are three people in a particular classroom, Alice, Bob, and Charlie. The statement "there is a student in the classroom" can be expressed as "Alice or Bob or Charlie is a student in the classroom". The statement "there is a student in the classroom" is false, if **everyone in the classroom**, i.e., Alice, Bob, and Charlie **is not** a student.

Truth Value of a Quantified Predicate

	$\forall x\ P(x)$	$\exists x\ P(x)$
true when	$P(x)$ **is true** for every x	There is an x for which $P(x)$ **is true**
false when	There is an x for which $P(x)$ **is false**	$P(x)$ **is false** for every x

Contents

Bounded and Free Variables

Bounded and Free Variables

Suppose P is a unary predicate, a variable x occurs in $P(x)$ is called *bounded variable* if

1. x is replaced by a particular element in domain D, or
2. x is bounded by a particular quantifier ($\forall x$ or $\exists x$)

A variable that is not bounded is called *free variable*. The terminology concerning bounded and free variables are not only for unary predicate, but also for other predicates with arity $n > 1$.

Example

Suppose P is a binary predicate, $P(x, y)$ is evaluated in domain $D_1 \times D_2$, we have:

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Example

Suppose Q is a ternary predicate, $Q(x, y, z)$ is evaluated in domain $D_1 \times D_2 \times D_3$, we have:

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- in $\exists x \in D_1 \; \exists y \in D_2 \; \forall z \in D_3 \; Q(x, y, z)$ we have x, y, and z are bounded variables
- in $Q(d_1, y, d_3)$ we have x and z are bounded variables (variables x and z are respectively replaced by d_1 and d_3), y is a free variable

Formula with Nested Quantifier

Let P be a ternary predicate whose universe of discourse is $D_1 \times D_2 \times D_3$. When D_1, D_2, and D_3 are clear from context, then the formula

$$\forall x \in D_1 \ \exists y \in D_2 \ \forall z \in D_3 \ P(x, y, z)$$

can be simplified as

$$\forall x \exists y \forall z \ P(x, y, z)$$

This rule is also applied to any predicate with arity $n > 1$.

In other words, we may omit writing the domain whenever the domain is clear from context.

Occurrence Order of (Nested) Quantifiers

The occurrence order of quantifiers can affect the meaning of predicate formulas.

Example

Suppose $\text{Teach}\,(x, y)$ means "person x teaches subject y" where the domain of x is the set of all lecturers in Tel-U and the domain of y is the set of all courses in Tel-U, then $\forall x \exists y\ \text{Teach}\,(x, y)$, $\exists y \forall x\ \text{Teach}\,(x, y)$, $\exists x \forall y\ \text{Teach}\,(x, y)$, $\forall y \exists x\ \text{Teach}\,(x, y)$ have different meanings:

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1. $\forall x \exists y\ \mathrm{Teach}\,(x, y)$ means "for every lecturer x, he/she teaches a subject y" or in other words

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1. $\forall x \exists y\ \text{Teach}(x, y)$ means "for every lecturer x, he/she teaches a subject y" or in other words "every lecturers in Tel-U at least teaches one subject",

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2. $\exists y \forall x\ \text{Teach}\,(x, y)$ means "there is a subject y which is taught by every lecturer x"

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Suppose $\text{Teach}(x, y)$ means "person x teaches subject y" where the domain of x is the set of all lecturers in Tel-U and the domain of y is the set of all courses in Tel-U, then $\forall x \exists y\ \text{Teach}(x, y)$, $\exists y \forall x\ \text{Teach}(x, y)$, $\exists x \forall y\ \text{Teach}(x, y)$, $\forall y \exists x\ \text{Teach}(x, y)$ have different meanings:

1. $\forall x \exists y\ \text{Teach}(x, y)$ means "for every lecturer x, he/she teaches a subject y" or in other words "every lecturers in Tel-U at least teaches one subject",
2. $\exists y \forall x\ \text{Teach}(x, y)$ means "there is a subject y which is taught by every lecturer x" or in other words "there is a subject taught by every lecturer in Tel-U",
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4. $\forall y \exists x\ \mathrm{Teach}\,(x, y)$ means "for every subject y, there is a lecturer x who teaches y" or in other words "every subject in Tel-U is taught by someone".

Quantifier Scope

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In predicate logic expression $\forall x \exists y \ \mathrm{Teach}\,(x, y)$ we have

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Contents

Precedence of Quantifiers and Other Operators

Suppose we have a logical expression $\forall x \ P(x) \wedge Q(x)$. We need to put the parentheses to make the expression clear, which one is correct?

1. $\forall x \ \left(P(x) \wedge Q(x)\right)$
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1. $\forall x \; (P(x) \wedge Q(x))$
2. $(\forall x \; P(x)) \wedge Q(x)$

In predicate logic, the quantifier $\forall$ and $\exists$ bind more tightly than other logical operators.

The precedence of quantifier and logical operators in predicate logic is described by following table:

Operator	Precedence
$\forall$	1
$\exists$	2
$\neg$	3
$\wedge$	4
$\vee$	5
$\oplus$	6
$\rightarrow$	7
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Therefore $\forall x\ P(x) \wedge Q(x)$ means $(\forall x\ P(x)) \wedge Q(x)$.

Contents

Terms in Predicate Logic

Predicate formulas are made up of terms which are defined as follows:

Terms

1. Any variable is a term. Variables are usually denoted by lowercase letters: u, v, w, x, y, z, $u_1, u_2, \ldots$, $v_1, v_2, \ldots$, $w_1, w_2, \ldots$, $x_1, x_2, \ldots$, $y_1, y_2, \ldots$, $z_1, z_2, \ldots$.
2. All constants in the domain (or universe of discourse) are terms. Constants are usually denoted by lowercase letters: a, b, c, $a_1, a_2, \ldots$, $b_1, b_2, \ldots$, $c_1, c_2, \ldots$, or concretely. For example, constants might be numbers $0, 1, 2$ (if the domain is a particular set of numbers), constants might be names $Alex$, Bob, or $Charlie$ (if the domain is a particular set of people), etc.
3. If $t_1, t_2, \ldots, t_n$ are terms and f is a function with arity $n \geq 1$, then $f\,(t_1, t_2, \ldots, t_n)$ is also term. In this case, f is considered as a function with n variable whose value is a single term.

Example

Suppose f is a unary function and g is a binary function, a and b are constants, x and y are variables, then:

1. a, b, x, and y are terms.
2. $f(a)$, $f(b)$, $f(x)$, $f(y)$ are terms, because f is a unary function.
3. $g(a, b)$, $g(y, x)$, $g(b, y)$, $g(x, x)$,

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5. $f(f(a))$, $f(f(b))$, $f(f(c))$

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Example

Suppose $0, 1, 2 \ldots$ are constants, x, y, z are variables, s is a unary function, $+$ and $\times$ are binary functions, then

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The terms $+(1, 2)$, $+(1, s(x))$, and $+(s(1), s(0))$ are usually written in infix notation, respectively as: $1 + 2$, $1 + s(x)$, and $s(1) + s(0)$.

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1. A term t is a subterm of t itself.
2. If s and t are two terms used for constructing more complex term u, then s and t are proper subterm of u.
3. Subterm is transitive: if s is a subterm of t and t is a subterm of u, then s is subterm of u.

Example

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Suppose 1 and 2 are constants, x is a variable, f is a unary function, and $+$ and $\times$ are binary function. Let t be a term $1 + (2 \times f(x))$, then the subterm of t are (1) $1 + (2 \times f(x))$, (2) $2 \times f(x)$,

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1. A term t is a subterm of t itself.
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Suppose 1 and 2 are constants, x and y are variables, f is a unary function, and $+$ and $\times$ are binary function. Determine all subterms of $(1 + f(1)) \times ((1 + x) \times (y + 2))$.

Solution:

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- $1 + f(1)$
- $(1 + x) \times (y + 2)$
- $1 + x$
- $y + 2$
- $f(1)$
- 1
- 2

Exercise

Suppose 1 and 2 are constants, x and y are variables, f is a unary function, and $+$ and $\times$ are binary function. Determine all subterms of $(1 + f(1)) \times ((1 + x) \times (y + 2))$.

Solution: subterms of $(1 + f(1)) \times ((1 + x) \times (y + 2))$ are:

- $(1 + f(1)) \times ((1 + x) \times (y + 2))$
- $1 + f(1)$
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Exercise

Suppose 2 is a constant, x and y are variables, s is a unary function, and $-$, $+$, $*$ are binary function. Determine all subterms of $(2 - s(x)) + (y * x)$.

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Solution: subterms of $(2 - s(x)) + (y * x)$ are:

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- $2 - s(x)$
- $y * x$

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- $2 - s(x)$
- $y * x$
- $s(x)$

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- $y * x$
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Parse Tree of A Term

Parse tree can be used to visualize the structure of a term in predicate logic. For example, if 2 is a constant, x and y are variables, s is a unary function, and $-$, $+$, $*$ are binary function, then the parse tree for term $(2 - (s(x) + y)) * x$ is

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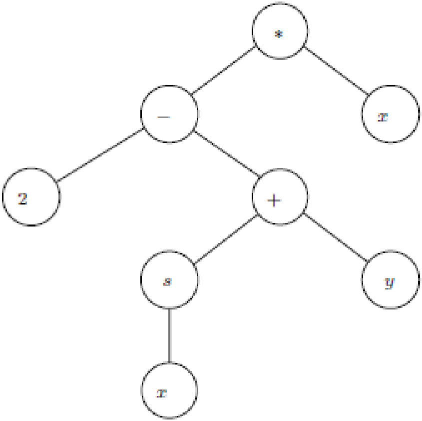

Predicate Formulas

Predicate Formulas

Formulas (or sentences) in predicate logic are made up of:

1. propositional constant: T (true) or F (false)
2. expression $P(t_1, t_2, \ldots, t_n)$ where $t_1, t_2, \ldots, t_n$ are terms and P is an n-ary predicate with $n \geq 1$
3. logical operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$

and comply following rules:

1. every well-defined expression $P(t_1, t_2, \ldots, t_n)$ is a predicate formula,
2. if A and B are two predicate formulas, then $\neg A$, $A \wedge B$, $A \vee B$, $A \oplus B$, $A \rightarrow B$, $A \leftrightarrow B$, are all predicate formulas,
3. if A is a predicate formulas and x is a variable, then both of $\forall x\ A$ and $\exists x\ A$ are predicate formulas.

Examples of Predicate Formulas

Example

According to the previous definition, if P, Q, R, S are predicates, then we have:

1. $\forall x P\left(x\right) \wedge Q\left(x\right)$ is a predicate formula, this formula can be written as $\left(\forall x P\left(x\right)\right) \wedge Q\left(x\right)$, variable x in $Q\left(x\right)$ is a free variable.
2. $\exists \forall x P\left(x\right) \vee Q\left(x, y\right)$ is not a predicate formula (because the expression $\exists \forall x$ is not well-defined).
3. $\forall x \exists P\left(x \rightarrow Q\left(x\right)\right)$

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1. $\forall x P(x) \wedge Q(x)$ is a predicate formula, this formula can be written as $(\forall x P(x)) \wedge Q(x)$, variable x in $Q(x)$ is a free variable.
2. $\exists \forall x P(x) \vee Q(x, y)$ is not a predicate formula (because the expression $\exists \forall x$ is not well-defined).
3. $\forall x \exists P(x \rightarrow Q(x))$ is not a predicate formula (because the expression $\exists P$ is not well-defined).
4. $\forall x \exists y \ (P(x, y) \rightarrow S(y, y))$

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4. $\forall x \exists y\ (P(x, y) \rightarrow S(y, y))$ is a predicate formula, which can be written as $\forall x (\exists y\ (P(x, y) \rightarrow S(y, y)))$.

Example

5. $\exists x \forall y \left(S\left(x, z\right) \wedge S\left(y, x\right)\right)$

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5. $\exists x \forall y\,(S(x,z) \wedge S(y,x))$ is a predicate formula, which can be written as $\exists x\,(\forall y\ \ (S(x,z) \wedge S(y,x)))$, variable z in $S(x,z)$ is a free variable.
6. $\forall x \forall y\,(P(x,y) \vee Q)$

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6. $\forall x \forall y \left(P\left(x, y\right) \vee Q\right)$ is not a predicate formula (because the expression Q without argument is not well-defined).
7. $\forall z \exists y \left(P\left(x\right) \rightarrow Q\left(y\right)\right)$

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7. $\forall z \exists y\,(P(x) \rightarrow Q(y))$ is a predicate formula, which can be written as $\forall z\,(\exists y\ \ (P(x) \rightarrow Q(y)))$, variable x in $P(x)$ is a free variable.
8. $P(x) \wedge (Q(x,y) \rightarrow \exists R\,(R(x)))$

Example

5. $\exists x \forall y \left(S\left(x,z\right) \wedge S\left(y,x\right)\right)$ is a predicate formula, which can be written as $\exists x \left(\forall y \ \left(S\left(x,z\right) \wedge S\left(y,x\right)\right)\right)$, variable z in $S\left(x,z\right)$ is a free variable.
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8. $P\left(x\right) \wedge \left(Q\left(x,y\right) \rightarrow \exists R\left(R\left(x\right)\right)\right)$ is not a predicate formula (because the expression $R\left(R\left(x\right)\right)$ is not well-defined).

Exercise

Suppose x and y are variables, a and b are constants over a particular domain D, f is a unary function over D, g is a binary function over D, P is a unary predicate, and Q is a binary predicate. Verify whether following expressions are well-defined predicate formulas.

1. $\forall x P\left(g\left(f\left(a\right), x\right)\right)$
2. $\exists x \forall y \left(P\left(x\right) \rightarrow Q\left(y, y\right)\right)$.
3. $\exists x \left(Q\left(x\right) \rightarrow P\left(x, y\right)\right)$.
4. $Q\left(a, g\left(f\left(a\right), f\left(b\right)\right)\right)$.
5. $P\left(a, f\left(x\right)\right)$.
6. $g\left(x, y\right) \rightarrow f\left(a\right)$.
7. $\exists x \forall y \left(f\left(x\right) \rightarrow g\left(x, y\right)\right)$.
8. $\forall x \left(P\left(x\right) \rightarrow g\left(a, f\left(x\right)\right)\right)$.
9. $\exists y \left(Q\left(y, y\right) \leftrightarrow P\left(y\right)\right)$.
10. $\exists y \exists x \left(Q\left(y, x\right) \wedge P\left(g\left(x, y\right)\right) \rightarrow P\left(a\right)\right)$.

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2. $\exists x \forall y \left(P\left(x\right) \rightarrow Q\left(y, y\right)\right)$. Predicate formula.
3. $\exists x \left(Q\left(x\right) \rightarrow P\left(x, y\right)\right)$. Not a predicate formula.
4. $Q\left(a, g\left(f\left(a\right), f\left(b\right)\right)\right)$. Predicate formula.
5. $P\left(a, f\left(x\right)\right)$. Not a predicate formula.
6. $g\left(x, y\right) \rightarrow f\left(a\right)$. Not a predicate formula.
7. $\exists x \forall y \left(f\left(x\right) \rightarrow g\left(x, y\right)\right)$. Not a predicate formula.
8. $\forall x \left(P\left(x\right) \rightarrow g\left(a, f\left(x\right)\right)\right)$. Not a predicate formula.
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10. $\exists y \exists x \left(Q\left(y, x\right) \wedge P\left(g\left(x, y\right)\right) \rightarrow P\left(a\right)\right)$. Predicate formula.

Subformula

The definition of subformula in predicate logic is analogous to the definition of subformula in propositional logic.

Subformula

1. A formula A is a subformula of A itself.
2. If A and B are two propositional formulas used to construct a more complex propositional formula C, then A and B are proper subformulas of C.
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Example

Let A be a formula $\forall x \exists y \ (P(x) \wedge Q(y,z) \rightarrow R(x,z))$, then the subformula of A are (1)

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3. Subformula is transitive: if A is a subformula of B and B is a subformula of C, then A is a subformula of C.

Example

Let A be a formula $\forall x \exists y\ (P(x) \wedge Q(y,z) \rightarrow R(x,z))$, then the subformula of A are (1) $\forall x \exists y\ (P(x) \wedge Q(y,z) \rightarrow R(x,z))$, (2) $\exists y\ (P(x) \wedge Q(y,z) \rightarrow R(x,z))$, (3) $P(x) \wedge Q(y,z) \rightarrow R(x,z)$, (4) $P(x) \wedge Q(y,z)$, (5) $P(x)$, (6) $Q(y,z)$, and (7) $R(x,z)$.

Exercise

Let m be a constant in the observed domain, determine all subformulas of $\forall x \forall y \ \ (F(x,m) \wedge S(y,x) \to B(x,m))$.

Solution:

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- $F(x, m) \wedge S(y, x) \rightarrow B(x, m)$

Exercise

Let m be a constant in the observed domain, determine all subformulas of $\forall x \forall y \ \left(F\left(x,m\right) \wedge S\left(y,x\right) \rightarrow B\left(x,m\right)\right)$.

Solution: subformulas of $\forall x \forall y \ \left(F\left(x,m\right) \wedge S\left(y,x\right) \rightarrow B\left(x,m\right)\right)$ are:

- $\forall x \forall y \ \left(F\left(x,m\right) \wedge S\left(y,x\right) \rightarrow B\left(x,m\right)\right)$
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- $F\left(x,m\right) \wedge S\left(y,x\right) \rightarrow B\left(x,m\right)$
- $F\left(x,m\right) \wedge S\left(y,x\right)$

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- $F(x,m) \wedge S(y,x)$
- $F(x,m)$

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- $F\left(x, m\right) \wedge S\left(y, x\right) \rightarrow B\left(x, m\right)$
- $F\left(x, m\right) \wedge S\left(y, x\right)$
- $F\left(x, m\right)$
- $S\left(y, x\right)$

Exercise

Let m be a constant in the observed domain, determine all subformulas of $\forall x \forall y \ (F(x,m) \wedge S(y,x) \rightarrow B(x,m))$.

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- $F(x,m) \wedge S(y,x)$
- $F(x,m)$
- $S(y,x)$
- $B(x,m)$

Exercise

Determine all subformulas of $\exists x\ \ (\exists z\ P(y,z) \wedge \forall y\ \ (\neg Q(y,x) \vee \exists z\ \neg P(y,z)))$.

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Solution: subformulas of $\exists x \ \left(\exists z \ P\left(y,z\right) \wedge \forall y \ \left(\neg Q\left(y,x\right) \vee \exists z \ \neg P\left(y,z\right)\right)\right)$ are:

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- $\exists z \ P\left(y,z\right) \wedge \forall y \ \left(\neg Q\left(y,x\right) \vee \exists z \ \neg P\left(y,z\right)\right)$
- $\exists z \ P\left(y,z\right)$

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- $\exists z\ P\left(y,z\right) \wedge \forall y\ \left(\neg Q\left(y,x\right) \vee \exists z\ \neg P\left(y,z\right)\right)$
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- $\exists z\ \neg P\left(y,z\right)$

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- $\neg Q(y,x) \vee \exists z \ \neg P(y,z)$
- $\exists z \ \neg P(y,z)$
- $\neg P(y,z)$

Exercise

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- $\neg P\left(y,z\right)$
- $\neg Q\left(y,x\right)$

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- $\neg Q(y,x) \vee \exists z\ \neg P(y,z)$
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- $\neg Q(y,x)$
- $P(y,z)$

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- $\exists z \ \neg P\left(y,z\right)$
- $\neg P\left(y,z\right)$
- $\neg Q\left(y,x\right)$
- $P\left(y,z\right)$
- $Q\left(y,x\right)$

Parse Tree of a Formula

Parse tree is useful to visualize the structure of a predicate formula.
For example, the parse tree of $\forall x \ ((P(x) \to Q(x)) \land S(x, y))$ is

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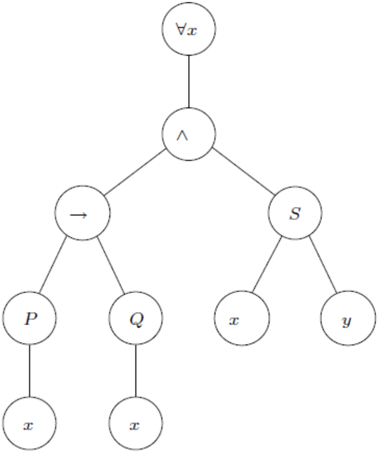

Exercise

Let x, y, z be variables, a be a constant, f be a unary function, and B, E, M, S be predicates. Draw the parse tree of each of these formulas.

1. $\forall x \exists y \ \left(M\left(x,y\right) \wedge \forall z \ \left(M\left(z,y\right)\right) \rightarrow E\left(x,z\right)\right)$.
2. $\forall x \exists y \forall z \ \left(M\left(x,y\right) \wedge \left(M\left(z,y\right) \rightarrow E\left(x,z\right)\right)\right)$.
3. $\forall x \left(\exists y \ S\left(x, f\left(y\right)\right) \rightarrow B\left(x, f\left(a\right)\right)\right)$.