Relation 1: Definition and Representations

Discrete Mathematics – Second Term 2022-2023

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Acknowledgements

This slide is composed based on the following materials:

1. *Discrete Mathematics and Its Applications*, 8th Edition, 2019, K. H. Rosen (primary).
2. *Discrete Mathematics with Applications* , 5th Edition, 2018, S. S. Epp.
3. *Mathematics for Computer Science*. MIT, 2010, E. Lehman, F. T. Leighton, A. R. Meyer.
4. Slide for Matematika Diskret 2 (2012). Fasilkom UI, B. H. Widjaja.
5. Slide for Matematika Diskrit. Telkom University, B. Purnama.

Some of the pictures are taken from the above resources. This slide is intended for academic purpose at FIF Telkom University. If you have any suggestions/comments/questions related with the material on this slide, send an email to `<pleasedontspam>@telkomuniversity.ac.id`.

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Cartesian Product

Definition

Let A and B be two sets, Cartesian product of A and B is written as $A \times B$ and defined as $A \times B := \{(a,b) \mid a \in A, b \in B\}$. In this case, (a,b) is called an *ordered pair* or 2-tuple.

Definition

Ordered pairs (a,b) and (c,d) are equal iff $a = c$ **and** $b = d$.

Example

If $A = \{1,2\}$ and $B = \{a,b,c\}$ then

$$A \times B \quad =$$

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If $A = \{1,2\}$ and $B = \{a,b,c\}$ then

$$\begin{aligned} A \times B &= \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\} \text{ and} \\ B \times A &= \end{aligned}$$

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If $A = \{1, 2\}$ and $B = \{a, b, c\}$ then

$$\begin{aligned} A \times B &= \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\} \text{ and} \\ B \times A &= \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}. \end{aligned}$$

We can see that $A \times B \neq B \times A$, so, in general, Cartesian product is not commutative.

Definition

Let $A_1, A_2, \ldots, A_n$ be n sets, the Cartesian product of $A_1, A_2, \ldots, A_n$ is written as $A_1 \times A_2 \times \cdots \times A_n$ and defined as follows

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}.$$

In this case, $(a_1, a_2, \ldots, a_n)$ is called ordered n-tuple (or n-tuple for short).

If $A_1 = A_2 = \cdots = A_n = A$, we can write $A \times A \times \cdots \times A$ as A^n. Two n-tuple $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are equal iff $a_i = b_i$ for every $i = 1, 2, \ldots, n$.

Some Important Theorems

Theorem

For A and B sets, we have

1. $(a, b) \in A \times B \Leftrightarrow (a \in A) \wedge (b \in B)$,
2. $(a, b) \in A \times B \Leftrightarrow (b, a) \in B \times A$,
3. $A = \emptyset \Rightarrow A \times B = B \times A = \emptyset$,
4. $A \times B = B \times A \Leftrightarrow (A = B) \vee (A = \emptyset) \vee (B = \emptyset)$.

Proof

Exercise.

Theorem

Let A and B be two finite sets, then

$$|A \times B| = |A| \cdot |B|,$$

with $|A|, |B|, |A \times B|$ is cardinality of set A, B, and $A \times B$, respectively.

Proof

Exercise.

Theorem

If $A_1, A_2, \ldots, A_n$ are finite sets, then

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|,$$

with $|A_i|$ is cardinality of A_i for $1 \leq i \leq n$ and $|A_1 \times A_2 \times \cdots \times A_n|$ is cardinality of $A_1 \times A_2 \times \cdots \times A_n$.

Proof

Exercise.

Contents

Binary Relation

Definition

Let A and B be two sets, binary relation from A to B is a subset of $A \times B$.

- If R is a relation from A to B, $a \in A$, and $b \in B$, then we write aRb if $(a, b) \in R$.
- We can say that aRb stated a is related to b. See that $a \in A$ and $b \in B$.
- Let $a \not R b$ or $a \bar{R} b$ or $\neg (aRb)$ denote $(a, b) \notin R$, or a is not related to b.
- Relation on A is relation from A to A. We have that relation on A is a subset of $A \times A$.

Domain and Range of Relation

Definition

Let A and B be two sets and R be a relation from A to B. Domain of R, denoted by $\mathrm{dom}\,(R)$, defined as

$$\mathrm{dom}\,(R) := \{a \in A \mid \text{there exists } b \in B \text{ such that } aRb\}\,.$$
$$\mathrm{dom}\,(R) := \{a \in A \mid \exists b \in B\,(aRb)\}\,.$$

In other words, $\mathrm{dom}\,(R)$ is a set containing all elements in A related to at least one element in B.
Range of R, denoted by $\mathrm{ran}\,(R)$, defined as

$$\mathrm{ran}\,(R) := \{b \in B \mid \text{there exists } a \in A \text{ such that } aRb\}\,.$$
$$\mathrm{ran}\,(R) := \{b \in B \mid \exists a \in A\,(aRb)\}\,.$$

To simplify, $\mathrm{ran}\,(R)$ is a set containing all elements in B such that at least one element in A is related to them.

Examples

Example

Let $A = \{Alex, Ben, Cathy\}$ is a set of students and $B = \{DM, C, DS, MVS\}$ is a set of courses. We have

$$A \times B = \left\{ \right.$$

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$$A \times B = \left\{ \begin{array}{l} (Alex, DM), (Alex, C), (Alex, DS), (Alex, MVS), \\ \\ \end{array} \right.$$

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$$A \times B = \left\{ \begin{array}{c} (Alex, DM), (Alex, C), (Alex, DS), (Alex, MVS), \\ (Ben, DM), (Ben, C), (Ben, DS), (Ben, MVS), \\ \end{array} \right.$$

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Example

Let $A = \{Alex, Ben, Cathy\}$ is a set of students and $B = \{DM, C, DS, MVS\}$ is a set of courses. We have

$$A \times B = \left\{ \begin{array}{c} (Alex, DM), (Alex, C), (Alex, DS), (Alex, MVS), \\ (Ben, DM), (Ben, C), (Ben, DS), (Ben, MVS), \\ (Cathy, DM), (Cathy, C), (Cathy, DS), (Cathy, MVS) \end{array} \right\}.$$

Let R be a relation from A to B defined as: "student x is taking y course" and we have these facts: Alex is taking DM and C, Ben is taking DM and DS, Cathy is taking DM and MVS.

Then

$$R = \left\{ \begin{array}{c} (Alex, DM), (Alex, C), \\ (Ben, DM), (Ben, DS), \\ (Cathy, DM), (Cathy, MVS) \end{array} \right\}.$$

- We can see that $R \subseteq A \times B$.
- $\mathrm{dom}\,(R) =$

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- $\mathrm{dom}\,(R) = \{Alex, Ben, Cathy\} = A$.
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- $\mathrm{dom}\,(R) = \{Alex, Ben, Cathy\} = A$.
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- $(Alex, DM) \in R$ or $Alex\ R\ DM$.
- $(Alex, MVS) \notin R$ or $Alex\ \not R\ MVS$ or $Alex\ \bar{R}\ MVS$ or $\neg\,(Alex\ R\ MVS)$.

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Let $A = \{1, 2\}$ and $B = \{x, y\}$. Let R be a relation from A to B with $R = \{(1, x), (2, x), (2, y)\}$, then $\operatorname{dom}(R) =$

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Example

Let $A = \{2, 3, 4\}$ and $B = \{2, 4, 8, 9, 15\}$. Let R be a relation from A to B defined as: aRb iff a divides b, for $a \in A$ and $b \in B$. Then

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Example

Let $\mathbb{Z}$ be the set of integers and R is a relation on $\mathbb{Z}$ defined as:

$$\text{for } a, b \in \mathbb{Z}, \text{ then } aRb \text{ iff } a = b^2.$$

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- $\text{ran}(R) = \{x \mid x \in \mathbb{Z}\} = \mathbb{Z}$.

Contents

Relation Representations of Relations over Finite Sets

If we have relations over finite sets, then we can represent those relations with:

1. arrow diagrams,
2. tables,
3. matrices, and
4. digraphs.

Arrow Diagrams

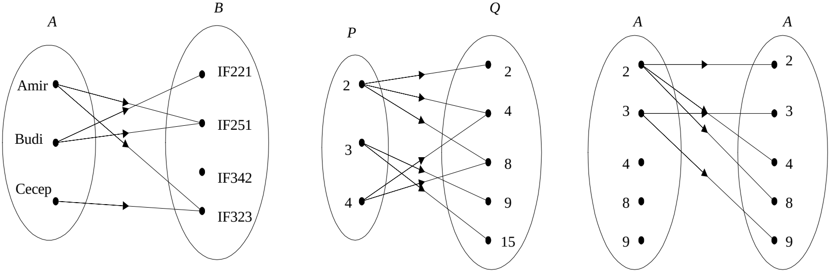
Let:

1. R_1 is a relation from $A = \{Amir, Budi, Cecep\}$ to $B = \{IF221, IF251, IF342, IF323\}$ with

$$R_1 = \left\{ \begin{array}{c} (Amir, IF251), (Amir, IF323), \\ (Budi, IF221), (Budi, IF251), (Cecep, IF323) \end{array} \right\}.$$

2. R_2 is a relation from $P = \{2, 3, 4\}$ to $Q = \{2, 4, 8, 9, 15\}$ with $R_2 = \{(2,2), (2,4), (2,8), (3,9), (3,15), (4,4), (4,8)\}$.
3. R_3 is a relation on $A = \{2, 3, 4, 8, 9\}$ with $R_3 = \{(2,2), (2,4), (2,8), (3,3), (3,9)\}$.

Arrow diagrams representation of R_1, R_2, and R_3 are:



Tables

Let R_2 be a relation from P to Q as defined above, then we can represent R_2 with the following table.

$\mathrm{dom}\,(R_2)$	$\mathrm{ran}\,(R_2)$
2	2
2	4
2	8
3	9
3	15
4	4
4	8

Representation Matrix of a Relation

Definition

Let $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be two non-empty finite sets and R is a relation from A to B. Relation R can be represented as an $|A| \times |B|$ matrix $\mathbf{M}_R$ defined as

$$\mathbf{M}_R = [m_{ij}], \text{ with } m_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$$\mathbf{M}_R = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccccc} b_1 & b_2 & \cdots & \cdots & b_n \end{array} \\ \left[\begin{array}{ccccc} m_{11} & m_{12} & \cdots & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ m_{m1} & m_{m2} & \cdots & \cdots & m_{mn} \end{array}\right] \end{array}$$

$\mathbf{M}_R$ is called a representation matrix of R.

Example

Exercise

Let $A = \{1, 2, 3\}$, $B = \{1, 2\}$, and R is a relation from A to B defined as: aRb iff $a > b$. Find a representation matrix of R if $a_1 = 1, a_2 = 2, a_3 = 3, b_1 = 1, b_2 = 2$.

Solution:

Example

Exercise

Let $A = \{1, 2, 3\}$, $B = \{1, 2\}$, and R is a relation from A to B defined as: aRb iff $a > b$. Find a representation matrix of R if $a_1 = 1, a_2 = 2, a_3 = 3, b_1 = 1, b_2 = 2$.

Solution: $R = \{(2, 1), (3, 1), (3, 2)\}$, then we have the representation matrix of R is

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Solution: $R = \{(2, 1), (3, 1), (3, 2)\}$, then we have the representation matrix of R is

$$\mathbf{M}_R = \left[\begin{array}{l} 0 \\ \\ \end{array} \right.$$

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Solution: $R = \{(2, 1), (3, 1), (3, 2)\}$, then we have the representation matrix of R is

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Digraph

Definition

Digraph (directed graph) is a graph contains set V of vertices and a set E in which its elements are ordered pairs of $V \times V$, which is called an edge. Vertex a is called *initial vertex* of edge (a, b), and b is the *terminal vertex* of edge (a, b). An edge of form (a, a) is called *loop*.

Digraph can only represent a relation on A, relation from A to B where $A \neq B$ cannot be represented as a digraph.

Exercise

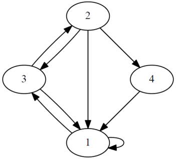
Draw a digraph representing relation
$R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$ in $\{1,2,3,4\}$.

Solution:

Exercise

Draw a digraph representing relation
$R = \{(1,1),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(4,1)\}$ in $\{1,2,3,4\}$.

Solution:



Digraph representing R.

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Draw a digraph representing relation
$R = \{(a,a), (a,b), (b,a), (b,c), (b,d), (c,a), (c,d), (d,b)\}$ in $\{a,b,c,d\}$.

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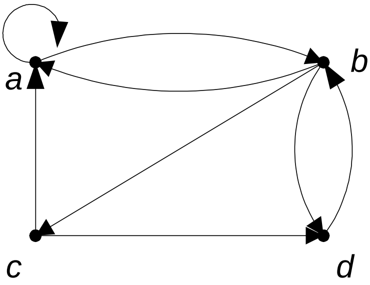
Draw a digraph representing relation
$R = \{(a,a), (a,b), (b,a), (b,c), (b,d), (c,a), (c,d), (d,b)\}$ in $\{a,b,c,d\}$.

Solution:



Contents

Set Operations on Relations

Definition

Let A and B be two sets, R, R_1, and R_2 be relations from A to B. We define

1. $R_1 \cup R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ or } (a,b) \in R_2\}$.
   $R_1 \cup R_2 = \{(a,b) \in A \times B \mid (aR_1b) \vee (aR_2b)\}$.
2. $R_1 \cap R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ and } (a,b) \in R_2\}$.
   $R_1 \cap R_2 = \{(a,b) \in A \times B \mid (aR_1b) \wedge (aR_2b)\}$.
3. $R_1 \oplus R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ or } (a,b) \in R_2, \text{ but not both}\}$.
   $R_1 \oplus R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \oplus (a,b) \in R_2\}$.
4. $R_1 \setminus R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ and } (a,b) \notin R_2\}$.
   $R_1 \setminus R_2 = \{(a,b) \in A \times B \mid (aR_1b) \wedge \neg (aR_2b)\}$.
5. $\neg R = \{(a,b) \in A \times B \mid (a,b) \notin R\}$. $\neg R$ can be written as $\bar{R}$
   $\neg R = \{(a,b) \in A \times B \mid \neg (aRb)\}$.
6. $R^{-1} = \{(b,a) \in B \times A \mid (a,b) \in R\}$.

Exercise

Let $A = \{a, b, c\}$ and $B = \{a, b, c, d\}$. If R_1 and R_2 are relations from A to B where:

$$\begin{aligned} R_1 &= \{(a,a),(b,b),(c,c)\} \\ R_2 &= \{(a,a),(a,b),(a,c),(a,d)\} \end{aligned}$$

Find:

1. $R_1 \cap R_2$
2. $R_1 \cup R_2$
3. $R_1 \oplus R_2$
4. $R_1 \setminus R_2$
5. $R_2 \setminus R_1$
6. $\neg R_1$ or $\bar{R}_1$
7. R_2^{-1}.

Solution:

1. $R_1 \cap R_2 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
3. $R_1 \oplus R_2 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
3. $R_1 \oplus R_2 = \{(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
4. $R_1 \setminus R_2 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a), (b,b), (c,c)\} \cap \{(a,a), (a,b), (a,c), (a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a), (a,b), (a,c), (a,d), (b,b), (c,c)\}$.
3. $R_1 \oplus R_2 = \{(a,b), (a,c), (a,d), (b,b), (c,c)\}$.
4. $R_1 \setminus R_2 = \{(b,b), (c,c)\}$.
5. $R_2 \setminus R_1 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
3. $R_1 \oplus R_2 = \{(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
4. $R_1 \setminus R_2 = \{(b,b),(c,c)\}$.
5. $R_2 \setminus R_1 = \{(a,b),(a,c),(a,d)\}$.
6. $\neg R_1 = \bar{R}_1 =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
3. $R_1 \oplus R_2 = \{(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
4. $R_1 \setminus R_2 = \{(b,b),(c,c)\}$.
5. $R_2 \setminus R_1 = \{(a,b),(a,c),(a,d)\}$.
6. $\neg R_1 = \bar{R}_1 = \{(a,b),(a,c),(a,d),(b,a),(b,c),(b,d),(c,a),(c,b),(c,d)\}$.
7. $R_2^{-1} =$

Solution:

1. $R_1 \cap R_2 = \{(a,a),(b,b),(c,c)\} \cap \{(a,a),(a,b),(a,c),(a,d)\} = \{(a,a)\}$.
2. $R_1 \cup R_2 = \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
3. $R_1 \oplus R_2 = \{(a,b),(a,c),(a,d),(b,b),(c,c)\}$.
4. $R_1 \setminus R_2 = \{(b,b),(c,c)\}$.
5. $R_2 \setminus R_1 = \{(a,b),(a,c),(a,d)\}$.
6. $\neg R_1 = \bar{R}_1 = \{(a,b),(a,c),(a,d),(b,a),(b,c),(b,d),(c,a),(c,b),(c,d)\}$.
7. $R_2^{-1} = \{(a,a),(b,a),(c,a),(d,a)\}$.

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow$

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Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow$

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Solution:

1. $(x,y) \in R_1 \cup R_2 \Leftrightarrow (x,y) \in R_1$ **or** $(x,y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 =$

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Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
2. $(x, y) \in R_1 \cap R_2 \Leftrightarrow$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

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Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

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3. $(x, y) \in R_1 \setminus R_2 \Leftrightarrow$

Exercise

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Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
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3. $(x, y) \in R_1 \setminus R_2 \Leftrightarrow (x, y) \in R_1$ **and** $(x, y) \notin R_2 \Leftrightarrow$

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3. $(x, y) \in R_1 \setminus R_2 \Leftrightarrow (x, y) \in R_1$ **and** $(x, y) \notin R_2 \Leftrightarrow x < y$ **and** $\neg (x > y) \Leftrightarrow x < y$ **and** $x \leq y \Leftrightarrow$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
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Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
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4. With the same reasoning in number 3, $R_2 \setminus R_1 =$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
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3. $(x, y) \in R_1 \setminus R_2 \Leftrightarrow (x, y) \in R_1$ **and** $(x, y) \notin R_2 \Leftrightarrow x < y$ **and** $\neg (x > y) \Leftrightarrow x < y$ **and** $x \leq y \Leftrightarrow x < y$. So $R_1 \setminus R_2 = R_1$.
4. With the same reasoning in number 3, $R_2 \setminus R_1 = R_2$.
5. $(x, y) \in R_1 \oplus R_2$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x, y) \in R_1 \cup R_2 \Leftrightarrow (x, y) \in R_1$ **or** $(x, y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$.
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3. $(x, y) \in R_1 \setminus R_2 \Leftrightarrow (x, y) \in R_1$ **and** $(x, y) \notin R_2 \Leftrightarrow x < y$ **and** $\neg(x > y) \Leftrightarrow x < y$ **and** $x \leq y \Leftrightarrow x < y$. So $R_1 \setminus R_2 = R_1$.
4. With the same reasoning in number 3, $R_2 \setminus R_1 = R_2$.
5. $(x, y) \in R_1 \oplus R_2 \Leftrightarrow (x, y) \in R_1 \cup R_2$ **and** $(x, y) \notin R_1 \cap R_2 \Leftrightarrow$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

Solution:

1. $(x,y) \in R_1 \cup R_2 \Leftrightarrow (x,y) \in R_1$ **or** $(x,y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x,y) \mid x \neq y\}$.
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4. With the same reasoning in number 3, $R_2 \setminus R_1 = R_2$.
5. $(x,y) \in R_1 \oplus R_2 \Leftrightarrow (x,y) \in R_1 \cup R_2$ **and** $(x,y) \notin R_1 \cap R_2 \Leftrightarrow x \neq y$ **and** $(x,y) \notin \emptyset \Leftrightarrow$

Exercise

Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

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1. $(x,y) \in R_1 \cup R_2 \Leftrightarrow (x,y) \in R_1$ **or** $(x,y) \in R_2 \Leftrightarrow x < y$ **or** $x > y \Leftrightarrow x \neq y$. So $R_1 \cup R_2 = \{(x,y) \mid x \neq y\}$.
2. $(x,y) \in R_1 \cap R_2 \Leftrightarrow (x,y) \in R_1$ **and** $(x,y) \in R_2 \Leftrightarrow x < y$ **and** $x > y$. Because it is not possible that $x < y$ and $x > y$ both happens, for all $x, y \in \mathbb{R}$, then $R_1 \cap R_2 = \emptyset$.
3. $(x,y) \in R_1 \setminus R_2 \Leftrightarrow (x,y) \in R_1$ **and** $(x,y) \notin R_2 \Leftrightarrow x < y$ **and** $\neg(x > y) \Leftrightarrow x < y$ **and** $x \leq y \Leftrightarrow x < y$. So $R_1 \setminus R_2 = R_1$.
4. With the same reasoning in number 3, $R_2 \setminus R_1 = R_2$.
5. $(x,y) \in R_1 \oplus R_2 \Leftrightarrow (x,y) \in R_1 \cup R_2$ **and** $(x,y) \notin R_1 \cap R_2 \Leftrightarrow x \neq y$ **and** $(x,y) \notin \emptyset \Leftrightarrow x \neq y$. So $R_1 \oplus R_2 =$

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Let R_1 be a relation on $\mathbb{R}$ defined as: xR_1y iff $x < y$. Let R_2 be a relation on $\mathbb{R}$ defined as: xR_2y iff $x > y$. Find all relations defined as: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \setminus R_2$, $R_2 \setminus R_1$, and $R_1 \oplus R_2$.

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Let R be a relation on $\mathbb{Z}$ defined as: xRy iff $x \leq y$. Find relation R^{-1} and relation $\bar{R}$ (or relation $\neg R$).

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Important Theorem

Theorem

Let A and B be two sets, R and S be relations from A to B, then

1. $\mathrm{dom}\left(R^{-1}\right) = \mathrm{ran}\left(R\right)$
2. $\mathrm{ran}\left(R^{-1}\right) = \mathrm{dom}\left(R\right)$
3. R^{-1} is a relation from B to A
4. $\left(R^{-1}\right)^{-1} = R$
5. $R \subseteq S$ iff $R^{-1} \subseteq S^{-1}$.